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Methods of Finding the LCM and GCD of Numbers

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Abstract: This article examines several methods for determining the Least Common Multiple (LCM) and Greatest Common Divisor (GCD) among natural numbers. The essence of each method, their application steps, as well as their advantages and disadvantages, are analyzed. Special attention is given to the Euclidean algorithm, the method through divisors, and the factorization method. Theoretical knowledge is reinforced through relevant examples, and recommendations that serve to develop students' mathematical thinking are provided.

Keywords: LCM, GCD, numbers, mathematical algorithm, divisor, multiple, Euclidean algorithm, arithmetic method, common divisor, common multiple.

Introduction

Mathematics is one of the main sciences that shapes consistent, precise, and logical thinking in humans. Each concept within it is interrelated and has deep logical meaning. When working with natural numbers, especially in studying their internal structure and mutual relations, the concepts of the Greatest Common Divisor (GCD) and the Least Common Multiple (LCM) are of particular importance. Through these two concepts, it becomes possible to determine how numbers are connected, the degree of their commonality, and the ratios between them.

The GCD and LCM are closely related and are widely used in mathematics to simplify complex problems, correctly determine ratios, and ensure equal distribution. Their application is especially important in simplifying fractions, dividing distances or time evenly, and standardizing measurements.

By deeply understanding this topic, not only is theoretical knowledge strengthened, but it also creates an opportunity to find clear solutions to many practical problems encountered in real life. There are several methods for finding the GCD and LCM, each based on different stages and logical approaches. Along with methods based on divisors and factors, efficient methods such as the Euclidean algorithm are also widely used. By learning these methods, students develop logical thinking and strengthen their mathematical skills.

Theoretical basis of the concepts of GCD and LCM

The Greatest Common Divisor (GCD) is the largest positive number that divides each of two or more natural numbers. The Least Common Multiple (LCM), on the other hand, is the smallest positive number that is a multiple of each of the given numbers. These concepts are fundamental elements of arithmetic and number theory, through which relationships and ratios between numbers are determined.

For example, let us take the numbers 12 and 18. Divisors of 12: 1, 2, 3, 4, 6, 12. Divisors of 18: 1, 2, 3, 6, 9, 18. Their common divisors: 1, 2, 3, 6. Thus, GCD (12, 18) = 6

For 4 and 6: Multiples of 4: 4, 8, 12, 16, 20, 24. Multiples of 6: 6, 12, 18, 24, 30

Common multiples: 12, 24, 36...Thus, LCM(4, 6) = 12

Methods of finding GCD and LCM

a) Through determining divisors

In this method, all divisors of each number are identified. Then, their common divisors are found, and the largest is selected — this is the GCD. To find the LCM, the multiples of each number are written in sequence, and the first common multiple is determined. This method is convenient for small numbers, but as the numbers grow, the method becomes inconvenient and time-consuming.

Example 1: Find GCD(20, 30)

Solution:

Divisors of 20: 1, 2, 4, 5, 10, 20

Divisors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Common divisors: 1, 2, 5, 10

GCD = 10

Example 2: Find LCM(8, 12)
Solution:
Multiples of 8: 8, 16, 24, 32, 40, 48...
Multiples of 12: 12, 24, 36, 48, 60...
Common multiples: 24, 48...
LCM = 24

b) Prime factorization method

Each number is factorized into prime factors. For GCD – only the smallest powers of the common prime factors are taken. For LCM – the highest power of each prime factor is taken.

Example:
$60 = 2^2 \times 3 \times 5$
$48 = 2^4 \times 3$
$GCD(60, 48) = 2^2 \times 3 = 12$
$LCM(60, 48) = 2^4 \times 3 \times 5 = 240$

This method is useful for reinforcing mathematical logic and is especially helpful when dealing with many numbers.

Example 3: Find GCD(72, 120)
Solution:
$72 = 2^3 \times 3^2$
$120 = 2^3 \times 3 \times 5$
Common prime factors: 2^3, 3 (only the smallest powers of common prime factors are taken)
$GCD = 2^3 \times 3 = 8 \times 3 = 24$

Example 4: Find LCM(18, 30)
Solution:
$18 = 2 \times 3^2$
$30 = 2 \times 3 \times 5$
All prime factors: 2, 3^2, 5 (the highest powers of each prime factor are taken)
$LCM = 2 \times 3^2 \times 5 = 90$

c) Euclidean algorithm

What is the significance of this property? This property allows replacing the given numbers with smaller ones when finding their GCD, simplifying the calculations. Such replacement can be performed several times.

1. The larger of the two numbers is divided by the smaller.
2. The remainder becomes the new number.
3. This process is repeated until the remainder is zero.
4. The remainder just before zero is the GCD.

Example:
Divide 525 by 231 with remainder, getting 63. So, D(525, 231) = D(231, 63)
Divide 231 by 63: 231 = 63•3 + 42 → D(231, 63) = D(63, 42)
Divide 63 by 42: 63 = 42•1 + 21 → D(63, 42) = D(42, 21)
Divide 42 by 21: remainder is 0 → D(42, 21) = D(21, 0)
The GCD of 21 and 0 is 21
Thus, the GCD of 525 and 231 is 21.
We often write calculations like this:
$525 = 231 \bullet 2 + 63$
$231 = 63 \bullet 3 + 42$
$63 = 42 \bullet 1 + 21$
$42 = 21 \bullet 2 + 0$
GCD(525, 231) = 21

This method of finding the greatest common divisor is based on division with remainder. It was first created by the ancient Greek mathematician Euclid (3rd century BC), hence it's known as the Euclidean algorithm. The general form of the Euclidean algorithm is as follows:

Let a and b be natural numbers and a > b. a is divided by b with remainder, then b is divided by the remainder, and then the first remainder is divided by the second remainder, and so on. In this case, the remainder different from zero at the last step is the GCD of a and b.

Example 5: Find GCD(105, 60) using Euclidean algorithm
Solution:
105 ÷ 60 = remainder 45
60 ÷ 45 = remainder 15
45 ÷ 15 = remainder 0
GCD = 15

This algorithm is also used in modern computer systems because it is fast, efficient, and easy to code.

d) Finding LCM through GCD

In this method, the product of two numbers is divided by their GCD:
$LCM(a, b) = (a \times b) / GCD(a, b)$
Example: a = 12, b = 18
GCD(12, 18) = 6
$LCM = (12 \times 18) / 6 = 216 / 6 = 36$

In this approach, it is enough to find the GCD; the LCM is automatically obtained.

The concepts of Greatest Common Divisor (GCD) and Least Common Multiple (LCM) are important foundations used not only in arithmetic, but also widely in algebra, number theory, algorithms, and cryptography. When determining these concepts, various methods — in particular, the Euclidean algorithm, the method of factorization, solving through linear expressions, and other modern algorithmic approaches — when studied comparatively, each has its own advantages and disadvantages.

Experience shows that for small numbers, the factorization method is sufficiently convenient and understandable, while for large numbers, the Euclidean algorithm stands out for its efficiency, simplicity, and speed. Especially from the perspective of computing technology and algorithmic complexity, the Euclidean method is used as an optimal solution in many modern programming languages. At the same time, understanding the formulas derived using the GCD in LCM calculation allows for deeper comprehension of mathematical relationships.

Analysis of theoretical sources and practical examples shows that studying this topic in depth is relevant not only at the general education level but also in academic and practical fields. Through such laws of commonality and harmony among numbers, students develop mathematical thinking, shape algorithmic reasoning, and become ready to use mathematical approaches in solving real-life problems.

As a result, it can be said that a deep understanding of methods for finding GCD and LCM lays a strong foundation for transitioning to other branches of mathematics. This serves as an essential basis for mastering even more complex mathematical concepts in the future.

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