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Research Article

APPROXIMATION IN A UNIFORM METRIC OF RANDOM PROCESSES BY TRIGONOMETRIC JACKSON POLYNOMIALS

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ABSTRACT

In the paper, we study the approximation of sub-Gaussian random processes (r.p.'s) by Jackson trigonometric polynomials.

KEYWORDS

sub-Gaussian random process, modulus of continuity, trigonometric Jackson polynomial, approximation.

INTRODUCTION

A random function $\xi(t)$, $t \in T \subset R^m$, $m \geq 1$ is said to be pre-Gaussian [3], [5] if there exist constants k and K ($0 < k, K < \infty$) such that $M exp\{k\xi(t)\} \leq K$.

Let a pre-Gaussian random function $\xi(t)$, $t \in T$ be such that $M\xi(t) = 0$, $\sup_{t\in T} \xi^2(t) > 0$. Then the function $\varphi(\lambda) = \max_{|x|=\lambda} \sup_{t\in T} lnMexp\{x\xi(t)\}$ is defined, continuous, monotonically increasing, and convex on $[0,\Lambda)$, for each $\lambda \in [0,\Lambda)$, there are left and right derivatives of the function $\varphi(\lambda)$, where $\Lambda = sup\{\lambda{:}\varphi(\lambda) < \infty\}$ [5]. In [5], it was also shown that the function $f(\lambda) = \frac{\varphi(\lambda)}{\lambda}$ is monotonically increasing on $[0,\Lambda)$, $\lim_{\lambda\to\infty} f(\lambda) = L$, $0 < L \leq \infty$, the function $\rho(t,s) = \sup_{x\neq 0} |x|^{-1}\chi(lnMexp\{x[\xi(t) - \xi(s)]\})$ is a semimetric on T, where $\chi(x)$ is the inverse function to $\varphi(\lambda)$. The metric ρ is called the natural metric of the function $\xi(t)$.

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Let (T, ρ) be the topological space corresponding to the metric ρ, $H(\varepsilon) = ln(\varepsilon)$ is the ε-entropy oft he space (T, ρ), where $N(\varepsilon)$ is the minimum possible number of points in the ε-network $S(\varepsilon)$ of the space (T, ρ).

Introduce the function $\Psi(\varepsilon) = \int_0^{\varepsilon} H(x)[\chi(H(x))]^{-1}dx$.

Theorem D [5]. Let $ξ(t)$, $t \in T$ be a pre-Gaussian, separable with respect to some set separable on (T, ρ), random function, $L = \infty$, $\Psi(\varepsilon) < \infty$. Then $ξ(t)$ is bounded, continuous on (T, ρ) with probability one, and for all $u \geq \inf_{p\in(0,1)} [\frac{2}{p(1-p)}\Psi(p) + \frac{1}{1-p}\varphi'(\frac{\lambda(H(p)-0)}{2(1-p)})]$, we have the estimate

$$P\{\sup_{t\in T} ξ(t) \geq u\} \leq exp\{-\varphi^*(u - \Psi^*(u))\}, \quad \text{где} \quad \Psi^*(u) = \inf_{p\in(0,1)} [up + \frac{2}{p}\Psi(p)]$$

where $\varphi^*(x) = \sup_{\lambda\geq 0} (\lambda x - \varphi(\lambda))$, $x \geq 0$ is the Young-Fenchel transformation [6].

A random variable (r.v.) $ξ$ is said to be sub-Gaussian [10] if there is $a \geq 0$ such that $Mexp\{ξ\lambda\} \leq \{\frac{a^2\lambda^2}{2}\}$ for all $\lambda \in R^1$.

Denote $\tau(ξ) = inf\{a \geq 0: Mexp\{ξ\lambda\} \leq \{\frac{a^2\lambda^2}{2}\}, \lambda \in R^1\}$.

It is known [2] that a r.v. $ξ$ is sub-Gaussian if and only if $Mξ = 0$ adn $\tau(ξ) < \infty$. It was also shown in [2] that $\tau(ξ) = \sup_{\lambda\neq 0}\{\frac{2lnMexp\{ξ\lambda\}}{\lambda^2}\}^{\frac{1}{2}}$, and the space of all sub-Gaussian r.v.'s $ξ$ with the norm $||ξ||_{sub} = \tau(ξ)$ is a Banach space.

A random function $ξ(t)$, $t \in T \subset R^m$ is said to be sub-Gaussian [2] if $Mξ(t) = 0$ and $\sup_{t\in T} \tau(ξ(t)) < \infty$.

Remark 1. Any sub-Gaussian random function $ξ(t)$, $t \in T$ is pre- Gaussian, and for it,

$$\varphi(\lambda) = \tau\cdot\frac{\lambda^2}{2}, \quad \chi(x) = \sqrt{\frac{2x}{\tau}}, \quad L = \infty, \quad \varphi^*(x) = \frac{x^2}{2\tau},$$

the natural metric $\rho(t,s) = \frac{1}{\sqrt{\tau}}||ξ(t) - ξ(s)||_{sub}$, where $\tau = \sup_{t\in T}||ξ(t)||_{sub}$.

Remark 2. Any centered Gaussian random function $ξ(t)$ is sub-Gaussian, and the norm $||ξ(t)||_{sub} = \{Mξ^2(t)\}^{\frac{1}{2}}$.

Theorem D implies the following estimate, which we will use in the future.

Corollary D. Let $ξ_0(t)$, $t \in T$, be a sub-Gaussian, separable with respect to some separable on (T, ρ_0) set, random function, where $\rho_0(t,s) = \frac{1}{\sqrt{\tau}}||ξ_0(t) - ξ_0(s)||_{sub}$, $t, s \in T$, $\tau = \sup_{t\in T}||ξ(t)||_{sub}$.

If $0 < \tau \leq 1$ и $\Psi(1) < \infty$, then, for all $u \geq 16\,\Psi(1)$,

$$P\{\sup_{t\in T} ξ_0(t) \geq u\} \leq exp\{-\frac{u^2 - 6u^{\frac{3}{2}}\sqrt{\Psi(1)}}{2}\}.$$

Proof of Corollary D. According to Remark 1, $L = \infty$, i.e., Theorem D is applicable for a sub-Gaussian random function $ξ_0(t)$. Since

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$$\inf_{p\in(0,1)} [\frac{2}{p(1-p)}\Psi(p) + \frac{1}{1-p}\varphi'(\frac{\lambda(H(p))}{2(1-p)} - 0)] = \inf_{p\in(0,1)} [\frac{2}{p(1-p)}\Psi(p) + \frac{\sqrt{2}}{2}\frac{\sqrt{\tau H(p)}}{(1-p)^2})],$$

and $\varphi^*(x) = \frac{x^2}{2\tau}$, then according to Theorem D,

$$P\{\sup_{t\in T} \xi_0(t) \ge u\} \le exp\{-\frac{u^2-2u\Psi^*(u) + [\Psi^*(u)]^2}{2\tau}\} \le exp\{-\frac{u^2-2u\Psi^*(u) + [\Psi^*(u)]^2}{2}\}$$

for all $u \ge \inf_{p\in(0,1)} [\frac{2}{p(1-p)}\Psi(p) + + \frac{\sqrt{2}}{2}\frac{\sqrt{\tau H(p)}}{(1-p)^2})]$.

Obviously,

$$\Psi(p) = \frac{\sqrt{2\tau}}{2}\int_0^p \sqrt{H(x)}\,dx \ge \frac{p\sqrt{2\tau H(p)}}{2}, \text{ i.e. } H(p) \le \Psi^2(p)\frac{2}{p^2\tau},$$

hence,

$$\inf_{p\in(0,1)} [\frac{2}{p(1-p)}\Psi(p) + \frac{\sqrt{2}}{2}\frac{\sqrt{\tau H(p)}}{(1-p)^2})] \le \inf_{p\in(0,1)} [\frac{2\Psi(p)}{p(1-p)} + \frac{\Psi(p)}{(1-p)^2})] \le 16\,\Psi\left(\frac{1}{2}\right) \le 16\,\Psi(1).$$

We obtain from here that, for all $u \ge 16\,\Psi(1)$,

$$P\{\sup_{t\in T} \xi_0(t) \ge u\} \le exp\{-\frac{u^2-2u\Psi^*(u) + [\Psi^*(u)]^2}{2\tau}\},$$

If we take into account that $\Psi^*(u) \le 6\sqrt{u\Psi(1)}$ and $exp\{-\frac{u^2-6u^{\frac{3}{2}}\sqrt{\Psi(1)}}{2}\} \le 1$ as $u \ge 16\,\Psi(1)$, then we come to the assertion of Corollary D. **Corollary D is proved.**

MAIN RESULTS

Let us consider a sub-Gaussian separable, measurable separable 2π- periodic mean-square continuous real sub-Gaussian r.p. $\xi(t)$, $t \in R^1$. Assume that the following condition is satisfied for it

(A): $||\xi(t) - \xi(s)||_{sub} \le \omega(|t-s|)$, $t, s \in R^1$,

where $\omega(z)$ is the modulus of continuity, for which there exists the inverse function $\omega^{-1}(x)$, and the integral $\int_0^1 \frac{\omega(z)}{z\sqrt{|lnz|}}dz < \infty$.

It is known [11], that the r.p. $\xi(t)$ is continuous with probability one.

We study the normalized process of deviations (n.p.d.) $\eta_n(t) = \frac{\xi(t) - D_n(\xi;t)}{C_0\omega(1/n)}$, where $D_n(\xi;t)$ is the Jackson operator (trigonometric polynomial):

$$D_n(\xi;t) = D_n\xi(t) = \int_{-\pi}^{\pi} \xi(t+x)\,D_n(x)dx = 2\pi\sum_{-(2n-2)}^{2n-2} \xi_k\,\varphi_k^{(n)}e^{ikt},$$

$D_n(x) = \frac{3}{2\pi(2n^2+1)n}\left(\frac{\sin\frac{nx}{2}}{\sin\frac{x}{2}}\right)^4$ is the Jackson kernel, $\mathfrak{z}_k$ and $\varphi_k^{(n)}$ are the Fourier coefficients of $\mathfrak{z}(t)$ and $D_n(x)$, respectively, $C_0 = \frac{\pi\sqrt{3}}{2} + 1$ is the Jackson constant [9, p.168]

Due to the 2π-periodicity of the n.p.d. $\eta_n(t)$, it suffices to study it on the interval $[-\pi, \pi]$.

Note that the n.p.d. $\eta_n(t)$ was studied in [8] when $\mathfrak{z}(t)$ is a stationary Gaussian r.p. and $\omega(x) = x^\alpha$, $0 < \alpha < 1$.

Theorem 1. If condition (A) is satisfied, then for $z \geq 64$, the inequality

$$P\{\max_{|t|\leq\pi}\left|\frac{\eta_n(t)}{\gamma_n}\right| < \frac{\sqrt{2}}{2}z\} \leq 2\exp\{-\frac{z^2}{16}\gamma_n^2\}$$

holds, where $\gamma_n = 2\sqrt{\ln n} + \frac{1}{\omega(1/n)}\int_0^{\frac{1}{n}}\frac{\omega(x)}{x\sqrt{|\ln x|}}dx + \sqrt{\ln(\pi+1)}.$

Proof of Theorem 1. We use Corollary D. To do this, we show that $\tau_n = ||\eta_n(t)||_{sub} \leq 1$ for all $n \in N$.

Indeed, for any $t \in [-\pi, \pi]$ and $n \in N$, we have

$$\tau_n = ||\eta_n(t)||_{sub} = \frac{1}{C_0\omega(1/n)}||\mathfrak{z}(t) - D_n(\mathfrak{z};t)||_{sub} \leq$$

$$\leq \frac{1}{C_0\omega(1/n)}\int_{-\pi}^{\pi}||\mathfrak{z}(t+x) - \mathfrak{z}(t)||_{sub}D_n(x)\,dx \leq$$

$$\leq \frac{1}{C_0\omega(1/n)}\int_{-\pi}^{\pi}\omega(|x|)D_n(x)\,dx \leq 1. \quad (1)$$

The last inequality follows from the Jackson theorem ([9], p. 167).

Obviously, $M\eta_n(t) = 0$, hence, by virtue of (1), the n.p.d. $\eta_n(t)$ is a sub-Gaussian r.p. for any $n \in N$.

Let $n \in N$ be any fixed one. Suppose that $\tau_n > 0$. (If $\tau_n = 0$, then $\eta_n(t) \equiv 0$ with probability one, and for this case, the assertion of Theorem 1 is obvious).

According to Remark 1, for $\eta_n(t)$, $\varphi_n(x) = \frac{\tau_n x^2}{2}$, $\chi(x) = \sqrt{\frac{2x}{\tau_n}}$, therefore, the natural metric $\rho_n(t,s) = \frac{1}{\sqrt{\tau_n}}||\eta_n(t) - \eta_n(s)||_{sub}$. For $\rho_n(t,s)$, $t, s \in [-\pi, \pi]$, we have

$$\rho_n(t,s) = \frac{1}{C_0\sqrt{\tau_n}\omega(1/n)}||\int_{-\pi}^{\pi}[\mathfrak{z}(t+x) - \mathfrak{z}(t) - \mathfrak{z}(s+x) + \mathfrak{z}(s)]\,D_n(x)dx||_{sub} \leq$$

$$\leq \frac{2\omega(|t-s|)}{C_0\sqrt{\tau_n}\omega(1/n)} \leq \frac{2\omega(|t-s|)}{\sqrt{\tau_n}\omega(1/n)}. \quad (2)$$

Using (2), we estimate the ε-entropy $H_n(\varepsilon)$ of the space $([-\pi, \pi], \rho_n)$.

Let $N_n(\varepsilon)$ be the minimum possible number of points in the ε-network of the set $[-\pi, \pi]$. Then inequality (2) implies that $N_n(\varepsilon) \leq M_n(\varepsilon)$, where

$$M_n(\varepsilon) = \min\{k \in N : \frac{2\omega(\pi/k)}{\sqrt{\tau_n}\omega(1/n)} \leq \varepsilon\},$$

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what implies

$$M_{\boldsymbol{n}}(\varepsilon) \leq \frac{\pi}{\omega^{-1}\left(\varepsilon \frac{\sqrt{\tau_n}}{2\omega(1/n)}\right)} + 1,$$

hence,

$$H_n(\varepsilon) = \ln N_n(\varepsilon) \leq \ln\left(\pi + \omega^{-1}\left(\frac{\varepsilon\sqrt{\tau_n}}{2\omega(1/n)}\right) + \ln\frac{1}{\omega^{-1}\left(\frac{\varepsilon\sqrt{\tau_n}}{2}\omega(1/n)\right)}\right.,$$

where $\omega^{-1}(x)$ is the function inverse to $\omega(x)$.

Estimate $\Psi_n(1) = \int_0^1 H_n(\varepsilon)\left[\chi_n\left(H_n(\varepsilon)\right)\right]^{-1} d\varepsilon$:

$$\Psi_n(1) = \frac{\sqrt{2\tau_n}}{2}\int_0^1 \sqrt{H_n(\varepsilon)}\, d\varepsilon \leq$$

$$\leq \frac{\sqrt{2\tau_n}}{2}\left\{\sqrt{ln\left[\pi + \omega^{-1}\left(\frac{\sqrt{\tau_n}}{2}\omega(1/n)\right)\right]} + \int_0^1 \sqrt{\left|ln\frac{1}{\omega^{-1}\left(\frac{\varepsilon\sqrt{\tau_n}}{2}\omega(1/n)\right)}\right|}\, d\varepsilon\right\} =$$

$$= \frac{\sqrt{2\tau_n}}{2}\left\{\sqrt{ln\left[\pi + \omega^{-1}\left(\frac{\sqrt{\tau_n}}{2}\omega(1/n)\right)\right]} + \frac{2}{\sqrt{\tau_n}}\int_0^{\frac{\sqrt{\tau_n}}{2}} \sqrt{\left|ln\frac{1}{\omega^{-1}(z\omega(1/n))}\right|}\, d z\right\}.$$

Using (1), we obtain from here that

$$\Psi_n(1) \leq \frac{\sqrt{2}}{2}\left\{\sqrt{ln(\pi+1)} + 2\int_0^1 \sqrt{\left|ln\frac{1}{\omega^{-1}(z\omega(1/n))}\right|}\, d z\right\} =$$

$$= \frac{\sqrt{2}}{2}\left\{\sqrt{ln(\pi+1)} + 2\sqrt{\ln n} + \frac{1}{\omega(1/n)}\int_0^{\frac{1}{n}} \frac{\omega(z)}{z\sqrt{|lnz|}}\, d z\right\},$$

i.e.

$$\Psi_n(1) \leq \frac{\sqrt{2}}{2}\gamma_n < \infty \text{ for each } n \in N. \qquad (3)$$

Obviously, the n.p.d. $\eta_n(t)$ is continuous with probability one, therefore [4, p. 203] it is separable on $([-\pi,\pi], \rho_0)$, where $\rho_0 = |t-s|$. By virtue of (2), the metric ρ_n is topologically equivalent to the metric ρ_0, therefore the n.p.d. $\eta_n(t)$ is separable on $([-\pi,\pi], \rho_n)$. Hence, taking into account (1), (3) and applying Corollary D, we obtain that for all $u \geq 36\Psi_n(1)$,

$$P\{\max_{|t|\leq\pi} \eta_n(t) \geq u\} \leq exp\{-\frac{u^2 - 6u^{\frac{3}{2}}\sqrt{\Psi_n(1)}}{2}\}.$$

From here, using (3), we arrive at the inequality

$$P\{\max_{|t|\leq\pi} \eta_n(t) \geq u\} \leq exp\{-\frac{u^2 - 6u^{\frac{3}{2}}\gamma_n\sqrt{\frac{\sqrt{2}}{2}}}{2}\} \text{ if } u \geq 18\,\gamma_n\sqrt{2}.$$

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Put $u = \frac{\sqrt{2}}{2}\gamma_n v$, then for $v \geq 36$,

$$P\{\max_{|t|\leq\pi}\frac{\eta_n(t)}{\gamma_n} \geq \omega(z)\, v\} \leq \exp\{(\frac{\sqrt{2}}{2}\gamma_n)^2(-\frac{v^2}{2}+3v^{\frac{3}{2}})\}.$$

If we assume that $v \geq 64$, then $v^2 - 6v^{\frac{3}{2}} \geq \frac{v^2}{4}$, therefore, for $v \geq 64$, the following inequality holds:

$$P\{\max_{|t|\leq\pi}\frac{\eta_n(t)}{\gamma_n} \geq \frac{\sqrt{2}}{2}\, v\} \leq \exp\{-\frac{v^2}{16}\gamma_n^2\}.$$

Finally, the inequality

$$P\{\sup_{|t|\leq\pi}|\frac{\eta_n(t)}{\gamma_n}| \geq \frac{\sqrt{2}}{2}\, v\} \leq\ 2P\{\sup_{|t|\leq\pi}\frac{\eta_n(t)}{\gamma_n} \geq \frac{\sqrt{2}}{2}\, v\}$$

implies the assertion of Theorem 1.

Theorem 1 is proved.

Corollary 1. Let $\mathcal{E} > 0$, $0 < \delta < 1$ and the conditions of Theorem 1 be satisfied.

If $\omega(1/n)\,\gamma_n \to 0$ as $n \to \infty$, then, for all $n \geq n_0 + 1$,

$$P\{\max_{|t|\leq\pi}|\mathfrak{Z}(t) - D_n(\mathfrak{Z};t)| < \mathcal{E}\} \geq 1-\delta,$$

where

$$n_0 = n_0(n_0,\delta) = \min\{n \in N: C_0\sqrt{2}\,\omega(1/n)\,(32\gamma_n + 2\sqrt{\ln\frac{2}{n}}) \leq \mathcal{E}\}.$$

Proof of Corollary 1. Put $z_0 = \frac{4}{\gamma_n}\sqrt{\ln\frac{2}{\delta}}$.

Then, according to Theorem 1,

$$P\{\max_{|t|\leq\pi}\frac{\eta_n(t)}{\gamma_n} \geq \frac{\sqrt{2}}{2}(64 + z_0)\} \leq\ 2\exp\{-\frac{(64+z_0)^2}{16}\gamma_n^2\} \leq 2\exp\{-\frac{z_0^2}{16}\gamma_n^2\},$$

$$\text{i.e. } P\{\max_{|t|\leq\pi}|\mathfrak{Z}(t) - D_n(\mathfrak{Z};t)| \geq\ C_0\sqrt{2}\,\omega(1/n)\,(32\gamma_n + 2\sqrt{\ln\frac{2}{n}})\} \leq \delta,$$

which proves Corollary 1.

Corollary 1 is proved.

Let $\mathfrak{Z}_0(t) \in C_\Omega^{2\pi}(R^1)$ be a Gaussian stationar r.p. with zero mean, unit variance and the continuous correlation function $r(t)$, satisfying the following condition [7], [8], [1]:

$$r(t) = 1-|t|^{2\alpha} + f(t),\quad 0 < \alpha \leq 1,\quad f(t) = o(|t|^{2\alpha}),\ \text{as } t \to 0. \tag{4}$$

According to Remark 2,

$$||\mathfrak{Z}_0(t) - \mathfrak{Z}_0(s)\,||_{sub} = \{M[\mathfrak{Z}_0(t) - \mathfrak{Z}_0(s)]^2\}^{\frac{1}{2}} = \{2[1 - r\,(t-s)]\}^{\frac{1}{2}},$$

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moreover, condition (4) implies that there exists a constant $C_1 > 0$ such that

$$\{2[1-r\,(t-s)]\}^{\frac{1}{2}} \leq C_1|t-s|^{\alpha},$$

i.e. the r.p. $ʒ_0(t)$ satisfies the condition of Theorem 1 with $\omega(x) = C_1|x|^{\alpha}$, $0 < \alpha \leq 1$,

$0 < C_1 < \infty$, and the condition of Corollary 1, hence, the following statement takes place.

Corollary 2. There is a constant C_1, $0 < C_1 < \infty$, such that for any $n \geq 3$, $0 < \delta < 1$, the inequality

$$\mathrm{P}\{\max_{|t|\leq\pi}|ʒ_0(t) - D_n\,(ʒ_0;t)| \geq C_0C_1\sqrt{2}\,[64\,n^{-\alpha}\sqrt{\ln n} + n^{-\alpha}(32\sqrt{\ln(\pi+1)} + 2\sqrt{ln\frac{2}{\delta}}) + \frac{32n^{-\alpha}}{\alpha}\mathcal{E}_n]\} \leq \delta$$

takes place, where $\mathcal{E}_n \sim \frac{1}{\sqrt{ln\,n}}$.

Proof of Corollary 2. The assertion of Corollary 2 follows from Theorem 1 if we take into account that

$$\gamma_n = \sqrt{ln(\pi+1)} + 2\sqrt{\ln n} + \frac{2n^{\alpha}}{\omega(1/n)}\int_{\sqrt{\alpha ln\, n}}^{\infty} \exp\{-u^2\}\, d\, z, \quad \text{когда } \omega(x) = C_1|x|^{\alpha}.$$

For comparison, we present one result from [8]:

Let $n \to \infty$ and $u = u(n) \to \infty$ such that $n = \left]\frac{\lambda}{2\pi\mu_\alpha(u)}\right[$,where $\lambda \in (0, \infty)$, $\mu_\alpha(u) = \frac{C_\alpha u^{\frac{2-2\alpha}{\alpha}}}{e^{-\frac{u^2}{2}}\sqrt{2\pi}}$, C_α is a constant depending only on α. We denote such a coordinated change in the level of u and n by $(n,u)_\alpha \to \infty$.

In [8], it is proved that $\lim_{n\to\infty} \sigma_n n^{-\alpha} = a_\alpha$ and, moreover, if the correlation function of the r.p. $ʒ_0(t)$ is such that $r''(t)|t|^{2-\alpha} = O(1)$, $t \to 0$, then

$$\lim_{(n,u)_\lambda \to \infty} P\{\max_{|t|\leq\pi}|ʒ_0(t) - D_n\,(ʒ_0;t)| > u\sigma_n\} = 1 - e^{-\lambda},$$

where $\sigma_n{}^2 = \{\mathrm{M}[ʒ_0(t) - D_n\,(ʒ_0;t)]^2\}^{\frac{1}{2}}$, a_α is a constant depending only on α .

These results imply that

$$\lim_{n\to\infty} P\{\max_{|t|\leq\pi}|ʒ_0(t) - D_n\,(ʒ_0;t)| > n^{-\alpha}b_\alpha\sqrt{ln\,n + \frac{1-\alpha}{\alpha}} + f_{\alpha,\lambda}(n)\} = 1 - e^{-\lambda},$$

where

$$0 < b_\alpha < \infty, \quad f_{\alpha,\lambda}(n) = o\,(n^{-\alpha}\sqrt{\ln n}),\, n \to \infty. \qquad (5)$$

Relation (5) and Corollary 2 show that, despite the generality of the considered class of r.p.'s, the estimate in Theorem 1 in specific cases is close to unimprovable in the sense of order in *n*.

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