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Research Article

SHARP ESTIMATES FOR THE APPROXIMATION OF PERIODIC RANDOM PROCESSES AND FIELDS BY JACKSON OPERATORS

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Shamshiev Abdivali
Associate Professors Of The Department Of General Mathematics Of The Jizzakh State Pedagogical University, Uzbekistan

ABSTRACT

In the work, we find sharp estimates for the root-mean-square error of the approximation of -periodic random processes and random fields by linear positive Jackson operators.

KEYWORDS

The Jackson trigonometric polynomial (operator), -periodic random process, random field, approximation, unimprovable inequality.

INTRODUCTION

The problem of approximation of uniformly continuous bounded nonrandom functions has a classical origin and has been known since the time of Newton. There are whole mathematical areas devoted to this theory, where the best approximations of continuous functions of a real and complex variable by interpolation and algebraic polynomials, trigonometric polynomials, approximations by splines, linear positive operators are studied, constructive characteristics of function classes are found, the cross-sections of function classes are estimated, and others [4]. [13], [14].

Methods of the approximation theory of non-random functions are also used in the study of problems of approximation of random functions, where well-studied, simple in construction algebraic and trigonometric polynomials, various interpolation formulas are chosen as the approximation apparatus. This approach is applied in works by Azlarov T.A. [1], Drozhzhina L.V. [5], [6], Kadyrova I.I. [7],

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Mirzakhmedov M.A., Khudaiberganov R. [8], Nagorny V.N., Yadrenko M.I. [9], Omarov S.O. [10], Seleznev O.V. [11], [12], Khudoyberganov R. [15] and others.

MAIN RESULTS

Denote by $C_{\Omega}^{2\pi}(R^1)$ the class of 2π-periodic, continuous with probability one random processes (r.p.'s), i.e. r.p.'s $\mathfrak{z}(t)$ such that $\mathfrak{z}(t)$ is continuous and $\mathfrak{z}(t+2\pi) = \mathfrak{z}(t)$ for any $t \in R^1$ with probability one.

Obviously, for almost all realizarions, $\mathfrak{z}(t) \in L_1(-\pi,\pi)$. Therefore, we can construct the following trigonometric Jackson polynomial [14]:

$D_n(\mathfrak{z};t) = D_n\mathfrak{z}(t) = \int_{-\pi}^{\pi} \mathfrak{z}(t+x)\, D_n(x)dx =$
$2\pi \sum_{-(2n-2)}^{2n-2} \mathfrak{z}_k\, \varphi_k^{(n)} e^{ikt}$ (1)

where $D_n(x) = \frac{3}{2\pi(2n^2+1)n}\left(\frac{\sin\frac{nx}{2}}{\sin\frac{x}{2}}\right)^4$ is the Jackson kernel, $\mathfrak{z}_k$ and $\varphi_k^{(n)}$ are the Fourier coefficients of $\mathfrak{z}(t)$ and $D_n(x)$, respectively.

$D_n\mathfrak{z}(t)$ is a linear and positive operator (l.p.o.). Consider the approximation of a r.p. $\mathfrak{z}(t) \in C_{\Omega}^{2\pi}(R^1)$ by the Jackson l.p.o. $D_n(\mathfrak{z};t)$.

Investigate the standard deviation

$\delta_n(\mathfrak{z};t) = \{M[\mathfrak{z}(t) - D_n(\mathfrak{z};t)]^2\}^{\frac{1}{2}}$.

Since the r.p. $\mathfrak{z}(t) - D_n(\mathfrak{z};t)$ is 2π-periodic, the function $\delta_n(\mathfrak{z};t)$ will be the same, so it suffices to study it on the interval $[-\pi,\pi]$.

Let $\omega_{\mathfrak{z}}(\delta) = \max_{|t-s|\le\delta}\{M[\mathfrak{z}(t) - \mathfrak{z}(s)]^2\}^{\frac{1}{2}}$ be the modulus of continuity of the r.p. $\mathfrak{z}(t)$.

Theorem 1. a) For any $\mathfrak{z}(t) \in C_{\Omega}^{2\pi}(R^1)$ and $n \in N$, the inequality

$$\max_{|t|\le\pi}\{M[\mathfrak{z}(t) - D_n(\mathfrak{z};t)]^2\}^{\frac{1}{2}} \le \left(\frac{4}{3} - \frac{45\sqrt{3}}{76\pi}\right)\omega_{\mathfrak{z}}\left(\frac{2\pi}{n}\right) \quad (2)$$

is valid.

b) inequality (2) is unimpovable for the class $C_{\Omega}^{2\pi}(R^1)$ in the sense that, for any ε > 0, there exist $\mathfrak{z}_\varepsilon(t) \in C_{\Omega}^{2\pi}(R^1)$ and $n_0 \in N$ such that

$$\max_{|t|\le\pi}\{M[\mathfrak{z}_\varepsilon(t) - D_{n_0}(\mathfrak{z}_\varepsilon;t)]^2\}^{\frac{1}{2}} > \left(\frac{4}{3} - \frac{45\sqrt{3}}{76\pi} - \varepsilon\right)\omega_{\mathfrak{z}_\varepsilon}\left(\frac{2\pi}{n_0}\right)$$

Proof of Theorem 1. First of all, we note that the Jackson kernel $D_n(x)$ has the following property: $\int_{-\pi}^{\pi} D_n(x)dx = 1$ for any $n \in N$ [14, p.79].

For any $\mathfrak{z}(t) \in C_{\Omega}^{2\pi}(R^1)$, $n \in N$, and $t \in [-\pi,\pi]$, uisng the above property of $D_n(x)$, the Fubini theorem, and the Cauchy-Bunyakovsky inequality, we have

$$\delta_n(\mathfrak{z};t) = \{M[\mathfrak{z}(t) - D_n(\mathfrak{z};t)]^2\}^{\frac{1}{2}} = \left\{M\left[\int_{-\pi}^{\pi}(\mathfrak{z}(t) - \mathfrak{z}(t+x))D_n(x)dx\right]^2\right\}^{\frac{1}{2}} \le$$
$$\le \int_{-\pi}^{\pi} \omega_{\mathfrak{z}}(|x|)\, D_n(x)dx\,.$$

Using the properties of the modulus of continuity $\omega_{\mathfrak{z}}(x)$, we obtain from here that

$$\delta_n(\mathfrak{z};t) \le 2\int_0^{\pi} \omega_{\mathfrak{z}}(x)\, D_n(x)dx \le 2\omega_{\mathfrak{z}}\left(\frac{2\pi}{n}\right)\int_0^{\pi}(1+]\frac{nx}{2\pi}[)\, D_n(x)dx = \omega_{\mathfrak{z}}\left(\frac{2\pi}{n}\right)\lambda_n,$$

where $\lambda_n = 2\int_0^{\pi}(1+]\frac{nx}{2\pi}[)\, D_n(x)dx$.

In [10], it is shown that $\sup_{n\ge1}\lambda_n = \lambda_3 = \frac{4}{3} - \frac{45\sqrt{3}}{76\pi} = 1{,}00688858\ldots$

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It follows from here that

$\delta_n(\xi;t) \leq \left(\frac{4}{3}-\frac{45\sqrt{3}}{76\pi}\right)\omega_{\xi}\left(\frac{2\pi}{n}\right)$

for any $\xi(t) \in C_{\Omega}^{2\pi}(R^1)$, $n \in N$, and $t \in [-\pi,\pi]$, hence,

$\max_{|t|\leq\pi}\delta_n(\xi;t) \leq \left(\frac{4}{3}-\frac{45\sqrt{3}}{76\pi}\right)\omega_{\xi}\left(\frac{2\pi}{n}\right)$.

Part a) of Theorem 1 is proved.

To prove part b) of Theorem 1, consider an even, 2π-periodic, non-random function defined on the interval $[0,\pi]$ in a following way:

$$f_\varepsilon(x)=\begin{cases} \frac{x}{\varepsilon} \text{ if } 0 \leq x \leq \varepsilon, \\ 1 \text{ if } \varepsilon \leq x \leq \frac{2\pi}{3}, \\ 1+\frac{1}{\varepsilon}\left(x-\frac{2\pi}{3}\right) \text{ if } \frac{2\pi}{3} \leq x \leq \frac{2\pi}{3}+\varepsilon, \\ 2 \text{ if } \frac{2\pi}{3}+\varepsilon \leq x \leq \pi \end{cases}$$

where ε is a sufficiently small number.

Let a r.v. ξ_0 be such that $M\xi_0^2 = 1$.

Obviously, the r.p. $\xi_\varepsilon(t) = \xi_0 f_\varepsilon(x) \in C_{\Omega}^{2\pi}(R^1)$ and $\omega_{\xi_\varepsilon}\left(\frac{2\pi}{3}\right) = 1$. For $\xi_\varepsilon(t)$, we have

$$\max_{|t|\leq\pi}\delta_3(\xi_\varepsilon;t) \geq \delta_3(\xi_\varepsilon;0) = \{M[\int_{-\pi}^{\pi}\xi_\varepsilon(x)D_3(x)\,dx]^2\}^{\frac{1}{2}} =$$

$$= \int_{-\pi}^{\pi} f_\varepsilon(x)D_3(x)\,dx = 2\{\int_0^{\varepsilon}\frac{x}{\varepsilon}\,D_3(x)\,dx + \int_{\varepsilon}^{\frac{2\pi}{3}} D_3(x)\,dx +$$

$$+ \int_{\frac{2\pi}{3}}^{\frac{2\pi}{3}+\varepsilon}[1+\frac{1}{\varepsilon}\left(x-\frac{2\pi}{3}\right)]\,D_3(x)\,dx + 2\int_{\frac{2\pi}{3}+\varepsilon}^{\pi} D_3(x)\} =$$

$$= 2[\int_0^{\frac{2\pi}{3}} D_3(x)\,dx + 2\int_{\frac{2\pi}{3}}^{\pi} D_3(x)\,dx] - 2[\int_0^{\varepsilon} D_3(x)\,dx - \int_0^{\varepsilon}\frac{x}{\varepsilon}\,D_3(x)\,dx\,] -$$

$$- 2[\quad 2\int_{\frac{2\pi}{3}}^{\frac{2\pi}{3}+\varepsilon} D_3(x)\,dx - \int_{\frac{2\pi}{3}}^{\frac{2\pi}{3}+\varepsilon}[1+\frac{1}{\varepsilon}\,(x-\frac{2\pi}{3})]\,D_3(x)\,dx] =$$

$$= 2\int_0^{\pi}\,(1+]\frac{2\pi}{3}[\,)D_3(x)\,dx - 2\int_0^{\varepsilon}\,(1-\frac{x}{\varepsilon})\,D_3(x)\,dx -$$

$$-2\int_{\frac{2\pi}{3}}^{\frac{2\pi}{3}+\varepsilon}[1+\frac{1}{\varepsilon}\left(x-\frac{2\pi}{3}\right)]\,D_3(x)\,dx = \lambda_3 - I_\varepsilon^{(1)} - I_\varepsilon^{(2)}$$

$$(\lambda_3 - I_\varepsilon^{(1)} - I_\varepsilon^{(2)})\,\omega_{\xi_\varepsilon}\left(\frac{2\pi}{3}\right).$$

It is obvious that

$I_\varepsilon^{(1)} \leq 2\int_0^{\varepsilon} D_3(x)\,dx \longrightarrow 0$, and $I_\varepsilon^{(2)} \leq 2\int_{\frac{2\pi}{3}0}^{\frac{2\pi}{3}+\varepsilon} D_3(x)\,dx \longrightarrow 0$ as $\varepsilon \longrightarrow 0$,

i.e.

$\max_{|t|\leq\pi}\delta_n(\xi_\varepsilon;t) \geq [\lambda_3-\alpha(\varepsilon)]\,\omega_{\xi_\varepsilon}\left(\frac{2\pi}{3}\right)$, $\alpha(\varepsilon) \longrightarrow 0$ as $\varepsilon \longrightarrow 0$

where $\alpha(\varepsilon) = I_\varepsilon^{(1)} + I_\varepsilon^{(2)}$.

This leads to the statement of Part b) of Theorem 1.

Theorem 1 is proved.

Theorem 2. For the class of r.p.'s $C_{\Omega}^{2\pi}(R^1)$, the relation

$$\sup_{n\in N,\xi\in C_{\Omega}^{2\pi}(R^1)} \frac{\max_{|t|\leq\pi}\{M|\xi(t)-Dn(\xi,t)|^2\}^{\frac{1}{2}}}{\omega_{\xi}\left(\frac{2\pi}{n}\right)} = \frac{4}{3}-\frac{45\sqrt{3}}{76\pi}$$

takes place.

Proof of Theorem 2 follows from Theorem 1.

Let us proceed to finding the sharp estimate for the approximation of random fields by the Jackson l.p.o.'s.

Denote by $C_{\Omega}^{2\pi}(R^2)$ the class of 2π-periodic in each argument and continuous with probability one r.p.'s $\xi(t,s)$.

The function

$$\omega_{\xi}^{(1)}(x_1,x_2) = \sup_{\substack{|t-t'|\leq x_1 \\ |s-s'|\leq x_2}}\{M|\xi(t,s)-\xi(t',s')|^2\}^{\frac{1}{2}},$$

$x_1, x_2 \geq 0$

is said to be the modulus of continuity of the first type of a r.p. $\mathfrak{Z}(t,s) \in C_{\Omega}^{2\pi}(R^2)$ [1], [5].

The function

$$\omega_{\mathfrak{Z}}^{(2)}(x) = \sup_{(t-t')^2+(s-s')^2 \le x^2} \{M|\mathfrak{Z}(t,s) - \mathfrak{Z}(t',s')|^2\}^{\frac{1}{2}}, x \ge 0$$

is said to be the modulus of continuity of the second type of a r.p. $\mathfrak{Z}(t,s) \in C_{\Omega}^{2\pi}(R^2)$.

The modules of continuity of a r.p. $\mathfrak{Z}(t,s) \in C_{\Omega}^{2\pi}(R^2)$ have the following properties:

1°. For any $0 \le x_1 \le x_1'$, $0 \le x_2 \le x_2'$, the inequalities

$$\omega_{\mathfrak{Z}}^{(1)}(x_1,x_2) \le \omega_{\mathfrak{Z}}^{(1)}(x_1',x_2) \le \omega_{\mathfrak{Z}}^{(1)}(x_1',x_2')$$

take place.

2°. $\omega_{\mathfrak{Z}}^{(1)}(nx_1,nx_2) \le n\omega_{\mathfrak{Z}}^{(1)}(x_1,x_2)$ for any $n \in N$, $0 \le x_1 \le x_2$.

3° $\omega_{\mathfrak{Z}}^{(2)}(x_1) \le \omega_{\mathfrak{Z}}^{(2)}(x_2)$ for any $0 \le x_1 \le x_2$.

4°. $\omega_{\mathfrak{Z}}^{(2)}(nx) \le n\omega_{\mathfrak{Z}}^{(2)}(x)$ for any $n \in N$, $0 \le x_1 \le x_2$.

5°. $\omega_{\mathfrak{Z}}^{(1)}(\frac{\sqrt{2}}{2}x,\frac{\sqrt{2}}{2}x) \le \omega_{\mathfrak{Z}}^{(2)}(x) \le \omega_{\mathfrak{Z}}^{(1)}(x,x) \le \omega_{\mathfrak{Z}}^{(2)}(x\sqrt{2})$, x ≥ 0.

Consider the approximation of a r.p. $\mathfrak{Z}(t,s) \in C_{\Omega}^{2\pi}(R^2)$ by the Jackson l.p.o.

$$D_{n,n}(\mathfrak{Z};t,s) = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\mathfrak{Z}(t+x,s+y)\,D_n(x)D_n(y)dxdy. \quad (3)$$

Theorem 3. a) For any $\mathfrak{Z}(t,s) \in C_{\Omega}^{2\pi}(R^2)$ and $n \in N$, the inequality

$$\max_{\substack{|t|\le\pi \\ |s|\le\pi}} \{M|\mathfrak{Z}(t,s) - D_{n,n}(\mathfrak{Z};t,s)|^2\}^{\frac{1}{2}} \le [2-(\frac{2}{3}-\frac{45\sqrt{3}}{76\pi})^2]\,\omega_{\mathfrak{Z}}^{(1)}(\frac{2\pi}{n},\frac{2\pi}{n}) \quad (4)$$

holds.

b) inequality (4) is unimprovable in the following sense: for any ε > 0, there exist $\mathfrak{Z}_\varepsilon(t,s) \in C_{\Omega}^{2\pi}(R^2)$ and $n_0 \in N$ such that

$$\max_{\substack{|t|\le\pi \\ |s|\le\pi}} \{M[\mathfrak{Z}_\varepsilon(t,s) - D_{n_0,n_0}(\mathfrak{Z}_\varepsilon;t,s)]^2\}^{\frac{1}{2}} > [2-(\frac{2}{3}-\frac{45\sqrt{3}}{76\pi})^2 - \varepsilon]\,\omega_{\mathfrak{Z}_\varepsilon}^{(1)}(\frac{2\pi}{n_0},\frac{2\pi}{n_0}).$$

Proof of Theorem 3. For any $\mathfrak{Z}(t,s) \in C_{\Omega}^{2\pi}(R^2)$, $n \in N$, and (t,s) ∈ $[-\pi,\pi]^2$, using the propertt of the Jackson kernels, the Fubini theorem and the Cauchy-Bunyakovsky inequality, we have

$$\delta_{n,n}(\mathfrak{Z};t,s) \equiv \{M[\mathfrak{Z}(t,s) - D_{n,n}(\mathfrak{Z};t,s)]^2\}^{\frac{1}{2}} =$$

$$= \{M[\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}[\mathfrak{Z}(t,s) - \mathfrak{Z}(t+x,s+y)]\,D_n(x)D_n(y)dxdy]^2\}^{\frac{1}{2}} \le$$

$$\le \int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\omega_{\mathfrak{Z}}^{(1)}(|x|,|y|)\,D_n(x)D_n(y)dxdy \le$$

$$\le \omega_{\mathfrak{Z}}^{(1)}(\frac{2\pi}{n},\frac{2\pi}{n})\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}(1+\max\{]\frac{n|x|}{2\pi}[,]\frac{n|y|}{2\pi}[\})\,D_n(x)D_n(y)dxdy =$$

$$= 4\omega_{\mathfrak{Z}}^{(1)}(\frac{2\pi}{n},\frac{2\pi}{n})\int_0^{\pi}\int_0^{\pi}(1+\max\{]\frac{n|x|}{2\pi}[,]\frac{n|y|}{2\pi}[\})\,D_n(x)D_n(y)dxdy = K_n\,\omega_{\mathfrak{Z}}^{(1)}(\frac{2\pi}{n},\frac{2\pi}{n})$$

where

$$K_n = 4\int_0^{\pi}\int_0^{\pi}(1+\max\{]\frac{n|x|}{2\pi}[,]\frac{n|y|}{2\pi}[\})\,D_n(x)D_n(y)dxdy.$$

It is obvious that $K_1 = K_2 = 1$. In [2], it is shownm that

$$\sup_{n\ge1} K_n = K_3 = 2-(\frac{2}{3}-\frac{45\sqrt{3}}{76\pi})^2 = 1{,}0137297\ldots.$$

It follows from here the proof of part a) of Theorem 2.

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To prove part b) of Theorem 2, consider an even 2π-periodic in each argument non-random function defined on $[0,\pi]^2$ as follows:

$$f_\varepsilon(t,s) = \left\{\begin{array}{ll} \frac{x}{\varepsilon} & \text{if } 0\le x\le\varepsilon,\ 0\le y\le x \\ \frac{y}{\varepsilon} & \text{if } 0\le y\le\varepsilon,\ 0\le x\le y \\ 1 & \text{if } \begin{cases}\varepsilon\le x\le\frac{2\pi}{3}, 0\le y\le x\\ \varepsilon\le y\le\frac{2\pi}{3}, 0\le x\le y\end{cases} \\ 1+\frac{1}{\varepsilon}\left(x-\frac{2\pi}{3}\right) & \text{if } \frac{2\pi}{3}\le x\le\frac{2\pi}{3}+\varepsilon,\ 0\le y\le x \\ 1+\frac{1}{\varepsilon}\left(y-\frac{2\pi}{3}\right) & \text{if } \frac{2\pi}{3}\le y\le\frac{2\pi}{3}+\varepsilon,\ 0\le x\le y \\ 2 & \text{if } \begin{cases}\frac{2\pi}{3}+\varepsilon\le x\le\pi,\ 0\le y\le x\\ \frac{2\pi}{3}+\varepsilon\le y\le\pi,\ 0\le x\le y\end{cases}\end{array}\right.$$

where ε > 0 is a sufficiently small number.

Let a r.v. ξ_0 be such that $M\xi_0^2 = 1$. Then the r.p. $\xi_\varepsilon(t,s) = \xi_0 f_\varepsilon(t,s) \in\ \in C_\Omega^{2\pi}(R^2)$ and $\omega_{\xi_\varepsilon}\left(\frac{2\pi}{3},\frac{2\pi}{3}\right) = 1$. We obtain from here that

$$\max_{\substack{|t|\le\pi\\|s|\le\pi}} \delta_{n,n}(\xi_\varepsilon;t,s) \ge \delta_{3,3}(\xi_\varepsilon;0,0)$$

$$= \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} f_\varepsilon(t,s)\,D_n(x)D_n(y)dxdy =$$

$$= 4\int_0^\pi\int_0^\pi f_\varepsilon(t,s)\,D_n(x)D_n(y)dxdy =$$

$$4\sum_{k=1}^4 \iint_{A_k} f_\varepsilon(t,s)\,D_3(x)D_3(y)dxdy \qquad (5)$$

where $A_1 = \{(x,y):\ 0\le x\le\varepsilon, 0\le y\le\varepsilon\}$,

$A_2 = \{(x,y): \varepsilon\le x\le\frac{2\pi}{3},\ 0\le y\le x\} \cup \{(x,y): \varepsilon\le x\le\frac{2\pi}{3}, 0\le x\le y\}$,

$A_3 = \{(x,y): \frac{2\pi}{3}\le x\le\frac{2\pi}{3}+\varepsilon, 0\le y\le x\} \cup \{(x,y): \frac{2\pi}{3}\le y\le\frac{2\pi}{3}+\varepsilon, 0\le x\le y\}$,

$A_4 = \{(x,y): \frac{2\pi}{3}+\varepsilon\le x\le\pi, 0\le y\le x\} \cup \{(x,y): \frac{2\pi}{3}+\varepsilon\le y\le\pi, 0\le x\le y\}$.

Taking into account definition of the function $f_\varepsilon(t,s)$, we have

$$\sum_{k=1}^4 \iint_{A_k} f_\varepsilon(t,s)\,D_3(x)D_3(y)dxdy =$$
$$\iint_{A_1} \max\{\tfrac{x}{\varepsilon},\tfrac{y}{\varepsilon}\}D_3(x)D_3(y)dxdy +$$
$$+\iint_{A_2} D_3(x)D_3(y)\,dxdy +$$
$$+\iint_{A_3} \max\{1+\tfrac{1}{\varepsilon}\left(x-\tfrac{2\pi}{3}\right), 1+\tfrac{1}{\varepsilon}\left(y-\tfrac{2\pi}{3}\right)\}\,D_3(x)D_3(y)dxdy +$$
$$+2\iint_{A_4} D_3(x)D_3(y)\,dxdy =$$
$$\iint_{A_1\cup A_2} D_3(x)D_3(y)\,dxdy +$$
$$+2\iint_{A_3\cup A_4} D_3(x)D_3(y)\,dxdy -- \iint_{A_1}(1-\max\{\tfrac{x}{\varepsilon},\tfrac{y}{\varepsilon}\})D_3(x)D_3(y)dxdy -$$
$$-\iint_{A_3}\left(2-\max\left\{1+\tfrac{1}{\varepsilon}\left(x-\tfrac{2\pi}{3}\right), 1+\tfrac{1}{\varepsilon}\left(y-\tfrac{2\pi}{3}\right)\right\}\right) D_3(x)D_3(y)dxdy \quad =\int_0^\pi\int_0^\pi(1+$$
$$\max\{]\tfrac{3x}{2\pi}[,]\tfrac{3y}{2\pi}[\})\,D_3(x)D_3(y)dxdy \quad - \quad \iint_{A_1}(1-\max\{\tfrac{x}{\varepsilon},\tfrac{y}{\varepsilon}\})D_3(x)D_3(y)dxdy -$$
$$-\iint_{A_3}(1+\tfrac{2\pi}{3\varepsilon}-\max\{\tfrac{x}{\varepsilon},\tfrac{y}{\varepsilon}\})\,D_3(x)D_3(y)dxdy \equiv \tfrac{1}{4}K_3 - I_\varepsilon^{(3)} - I_\varepsilon^{(4)} \ge \tfrac{1}{4}K_3 - |I_\varepsilon^{(3)}| - |I_\varepsilon^{(4)}|.$$

Obviously,

$|I_\varepsilon^{(3)}| \le \iint_{A_1} D_3(x)D_3(y)\,dxdy \to 0$ as $\varepsilon \to 0$,

$|I_\varepsilon^{(4)}| \le \iint_{A_3} D_3(x)D_3(y)\,dxdy \to 0$ as $\varepsilon \to 0$.

Thus,

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$\sum_{k=1}^{4} \iint_{A_k} f_\varepsilon(t,s)\, D_3(x) D_3(y)\, dxdy \geq K_3 - \theta(\varepsilon),\ \ \theta(\varepsilon) \to 0$ as $\varepsilon \to 0$.

Taking into account relation (5), we obtain from here that

$$\max_{\substack{|t|\le\pi \\ |s|\le\pi}} \delta_{n,n}(\xi_\varepsilon; t,s) \geq \delta_{3,3}(\xi_\varepsilon; 0,0) \geq K_3 - \theta(\varepsilon) = [K_3 - \theta(\varepsilon)]\, \omega^{(1)}_{\xi_\varepsilon}\left(\frac{2\pi}{3}, \frac{2\pi}{3}\right)$$

Let $\varepsilon_1 > 0$ be an arbitrary number. Choosng $\theta(\varepsilon)$ such that $\theta(\varepsilon) < \varepsilon_1$, we come to the assertion of the second part of Theorem 3.

Theorem 3 is proved.

Theorem 4. For the class of r.p.'s $C_\Omega^{2\pi}(R^2)$, the relation

$$\sup_{n\in N,\ \xi(t,s)\in C_\Omega^{2\pi}(R^1)} \frac{\max_{(t,s)\in R^2} \{ M|\xi(t,s) - Dn,n(\xi;t,s)|^2 \}^{\frac{1}{2}}}{\omega_\xi(\frac{2\pi}{n})} = \frac{4}{3} - \frac{45\sqrt{3}}{76\pi}$$

holds.

The proof of Theorem 4 follows from Theorem 3.

Note that Theorems 1–4 in the case when $\xi(t)$ and $\xi(t,s)$ are nonrandom functions coincide with the results of [2] obtained there by another method.

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