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MATHEMATICAL MODELS FOR IMPROVING TEACHER PRODUCTIVITY

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ABSTRACT

Analysis of variance is the identification and evaluation of individual factor features that determine the variability of an effective feature. The significance of various factors is determined by the calculation method. The average age, profitability, utilization rate, labor productivity per employee and average salary per person were significant. The analysis of variance also revealed the influence of a number of factors on the main socio-economic indicators of higher education institutions. The article describes a quantitative assessment of the factors of improving the efficiency of staff in higher education institutions.

KEYWORDS

Variance, regression, factors, variance analysis, higher education institutions, teacher labor efficiency.

INTRODUCTION

The purpose of analysis of variance is to test the significance of the difference between means by comparing variances. The dispersion of the measured trait is decomposed into independent terms, each of which characterizes the influence of a particular factor or their interaction. Subsequent comparison of such

terms makes it possible to assess the significance of each studied factor, as well as their combination [1].

Most of the phenomena and processes are in constant mutual and all-encompassing objective communication. The study of dependencies and interrelations between objectively existing

phenomena and processes plays an important role in various spheres of public life. It provides an opportunity to understand the complex mechanism of cause-effect relations between phenomena better. Correlation-regression analysis is widely used to study the intensity, type and form of dependencies, which is a methodological toolkit for solving problems of forecasting, planning and analyzing the activities of universities.

When studying and analyzing socio-economic factors affecting the effectiveness of teachers' work, one of the most important moments is to identify the significance of the influence of these or those factors in their total population on the effective indicator. In practice, this is very difficult. The task is easier if one can use the method of dispersion analysis, which is one of the sections of mathematical statistics.

Dispersion analysis is the identification and evaluation of individual factorial traits that cause the variability of the effective trait. Each factor trait varies in the total population of units. The accepted way of measuring and analyzing the variation of these features is the basis of the dispersion analysis as a method of studying the significance of factors.

The task of the dispersion analysis is to isolate the sign from the total variability of the sign:

- variability caused by the action of each of the investigated independent variables;

- variability caused by the interaction of the studied characteristics;
- random variability caused by all other unknown variables.

The idea of the dispersion analysis is to decompose the overall dispersion of the efficacious trait into the parts caused by the influence of controlled factors and the residual dispersion caused by uncontrolled influence or random circumstances. Conclusions about the materiality of the influence of controlled factors on the result are made by comparing the parts of the total dispersion when meeting the requirement of normal distribution of the resultant attribute.

Many dispersion analysis models are known. They are classified, on the one hand, by the mathematical nature of the factors (deterministic, random and mixed) and, on the other hand, by the number of controlled factors (single-factor and multi-factor models). Models with more than one factor make it possible to investigate not only the influence of individual controlled factors (main influences) on the result, but also their superposition (interactions). Complete and incomplete t-factor plans, complete and incomplete block plans and randomized block plans [2] stand out by the way of organizing the initial data among the models of dispersion analysis.

The object of the dispersion analysis study is stochastic relations between the response (reaction) and factors

when the latter are not quantitative but qualitative in nature [3]. The main idea of the dispersion analysis is to compare the "factor dispersion" generated by the influence of the factor and the "residual dispersion" caused by random causes [4]. If the difference between these dispersions is significant, the factor has a significant impact on X; in this case, the average values of observed values at each level (group averages) also differ significantly. If it has already been established that the factor significantly affects X, and it is necessary to find out which of the levels has the greatest impact, the comparison of averages is additionally made in pairs. There are two models of variance analysis:

- with fixed levels of factors,
- with random factors.

METHODS

Depending on the number of factors determining the variation of the effective feature, the dispersion analysis is subdivided into one-factor and multifactor.

The main schemes of the organization of the initial data with two or more factors are:

- cross-classification, which is characteristic of models with fixed levels of factors.
- hierarchical (nested) classification characteristic of random factor models.

A dispersion analysis is based on the division of the dispersion into parts or components. Intra-group dispersion explains the influence of factors not taken into account in grouping, and inter-group dispersion explains the influence of grouping factors on the group average.

Single-factor dispersion analysis can be used to identify the most significant links between variables in the qualitative study of objects of different nature. Single-factor dispersion analysis is used to compare average values for three or more samples.

As a disadvantage, it may be impossible to identify samples that are different from others. For this purpose, it is necessary to use the Sheffe method or to conduct paired comparisons of samples. In addition to the functions of single-factor dispersion analysis, the multifactor analysis evaluates the inter factor interaction [5].

In practice, it is often necessary to check the materiality of the difference between the sample averages of m samples ($m > 2$). For example, it is necessary to assess the influence of various factors on the efficiency of teachers' work, the increase of students' cognitive activity on the indicators of education quality, the improvement of mechanisms of the social management system on the studied indicators, etc. To solve this problem effectively, a new

approach is needed, which is implemented in the dispersion analysis.

As it was noted, dispersion analysis is a statistical method of analysis of test results, the purpose of which is to assess the impact of one or more qualitative factors on the considered value of X , as well as for subsequent planning. That is, it is the analysis of the variability of the attribute under the influence of controlled variables. In foreign literature, variance analysis is often referred to as ANOVA, which is

translated as Analysis of Variance. The author of the method is R.A. Fisher (Fisher R.A., 1918, 1938).

By the number of factors, the influence of which is studied, distinguishes between single-factor and multifactor dispersion analyses. The essence of the dispersion analysis consists in the dismemberment of the general dispersion of the studied feature on the separate components caused by the influence of concrete factors, and the testing of hypotheses about the significance of the influence of these factors on the investigated feature.

Table 1.

Basic concepts and formulas

		Dispersion Types
- group and intra group - inter group and general		D_{jgr} and D_{ngr}
		D_{ingr} and D_{gen}
Dispersion analyses		
	X – research factor	μ – overall average feedback Y ;
Y – feedback (experiment result) Model: $Y_{ij} = \mu + F_i + \varepsilon_{ij}$		F_i – factor influence X_i on Y ε_{ij} – random balance
Factor (intergroup) dispersion S^2_{fact} (X)	←comparison→ ↓ Factor influence degree	Residual (intergroup) dispersion S^2_{in}

The two-factor dispersion analysis is used when the simultaneous effect of two factors on different samples of objects is studied, i.e. when different samples are under the influence of different combinations of two factors. It can happen that one

variable has a significant effect on the studied feature only at certain values of another variable. The essence of the method remains the same as in the case of a single-factor model, but more hypotheses can be tested in the two-factor dispersion analysis [6].

The solution of the two-factor dispersion analysis problem depends on the number of observations made at each combination of factor levels, if, in other words, in each cell of the two-factor complex.

The dispersion analysis is designed to evaluate the influence of different but controlled factors on the result of the experiment. Let the result of the experiment be some random value of Y , also called a response. The values of the random value of Y are influenced by the factor X , consisting of n -levels. Depending on the number of factors included in the analysis, a distinction is made between single-factor, two-factor and multi-factor dispersion analysis.

A dispersion analysis is possible if the measurement results are independent random variables subject to the normal distribution law with the same dispersions. Single-factor dispersion analysis reveals the degree of influence of one factor X on the mathematical expectation of $M(Y)$ response. The factor can be quantitative or qualitative. In the course of the experiment, the factor X is supported at n -levels. At each level of the factor m duplicate experiments are carried out. Value m can be the same or different for each level. The results of all measurements are presented in the form of a table called an observation matrix [7].

Table 2.

Surveillance Matrix

Factor level number	Factor level	Observation	Number of duplicate experiments
1	X_1	$Y_{11}, Y_{12}, \dots, Y_{1j}, \dots, Y_{1m_1}$	m_1
...	...	...	...
i	X_i	$Y_{i1}, Y_{i2}, \dots, Y_{ij}, \dots, Y_{im_i}$	m_i
...	...	...	...
n	X_n	$Y_{n1}, Y_{n2}, \dots, Y_{nj}, \dots, Y_{nm_n}$	m_n
			n $N = \sum_{i=1} m_i$

First, for each series of duplicate experiments, the arithmetic mean μ_i is calculated, which are the estimations of $M(Y_i)$ and S^2_{bi} reproducibility dispersion (Tab.3).

Then the homogeneity of a series of dispersions S^2 is checked either in pairs using the Fisher criterion (if m_i are different) or using the Cochran criterion (if m_i are

constant). For this purpose, we formulate a null hypothesis $H_0: D(X_1) = D(X_2) = \dots = D(X_L)$. The observed value of the Kohren criterion is determined by samples of one volume:

$G_{obs} = S^2_{max} / \sum_{i=1}^L S^2_i$. The observed value of the criterion is compared with the critical point of the right critical region $G_{cr}(\alpha; k; L)$, where $k = m -$

1(Appendix 8) and the homogeneity of dispersions is concluded. If the dispersions are not homogeneous, no further analysis is carried out.

After confirming the hypothesis about the homogeneity of dispersions, you can proceed to the analysis. It is believed that the result of any measurement Y_{ij} can be represented by a model:

$$Y_{ij} = \mu + F_i + \varepsilon_{ij},$$

where Y_{ij} is the value of the studied variable obtained at the i -th level of the factor with the j -th serial number;

μ is the overall average of the response Y ;

F_i - the effect of the influence of factor X_i on Y : deviation of the mean values of μ_i at the i -th level (group means) from the general average μ (i.e $F_i = \mu_i - \mu$);

ε_{ij} is a random remainder reflecting the influence of all other uncontrolled (unaccounted)

factors on the value of Y_{ij} .

The main assumptions of ANOVA are as follows:

- the remainders ε_{ij} are mutually independent for any i and j ;
- the ε_{ij} values are subject to the normal law [8].

The task of analysis of variance is to assess the significance of the effect of changes in the level of a factor. The dispersion of the response values caused by the controlled factor is estimated by the factor variance (the sum of the squares of the deviations of the group means from the total mean) – $S^2_{fact}(X)$.

The influence of uncontrollable factors (contribution ε_{ij}) can be estimated by the average variance of reproducibility (residual variance) – S^2_{in} .

The total dispersion of the response values caused by both controlled and uncontrolled factors is estimated by the total (or total) variance (total sum of squares of deviations) – S^2_{total} .

Table 3.

Formulas for calculating dispersions in dispersion analysis

Average arithmetic (group)	Dispersion reproducibility (group)	Residual (intra-group) dispersion
$\mu_i = \frac{1}{m_i} \sum_{j=1}^{m_i} Y_{ij}$	$S^2_{in_i} = \frac{1}{m_i} \sum_{j=1}^{m_i} (Y_{ij} - \mu_i)^2$	$S^2_{in} = \frac{1}{n} \sum_{i=1}^n S^2_{in_i}$
Total average	Factor (intergroup) dispersion	Total (full) dispersion
$\mu = \frac{1}{n} \sum_{i=1}^n \mu_i$	$S^2_{fact}(X) = \frac{1}{n-1} \sum_{i=1}^n m_i (\mu_i - \mu)^2$	$S^2_{total} = \frac{1}{N-1} \sum_{i=1}^n \sum_{j=1}^{m_i} (Y_{ij} - \mu)^2$

CONCLUSION

So, in this article the following is done:

1. The quantitative assessment of the factors of increasing the efficiency of personnel at universities is characterized.
2. Mathematical models of socio-economic factors influencing the efficiency of teachers' work are proposed.
3. The method of calculation determines the importance of various factors: average age,

profitability, utilization factor, output of one employee and average salary per person.

4. The method of dispersion analysis established the influence of a number of factors on the main socio-economic indicators of universities.

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