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The Geometric Property Of Square Equations Equation When They Have Complex Solutions

Seydullaev Abat
PhD, Associate Professor, Karakalpak State University, Uzbekistan

Koshmuratova Gulnaza
2nd Grade Student, Faculty Of Mathematics, Karakalpak State University, Uzbekistan

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Abstract: The article discusses a technique for visualizing the solutions of square equations. Square equations can have any real coefficients, including those leading to complex solutions. We have also given geometric illustrations of two square functions constructed, as well as in the article presented when square equations have not valid solutions, surely they have complex (imaginary) solutions. In addition shown their graphs on the plane with real and imaginary axises.

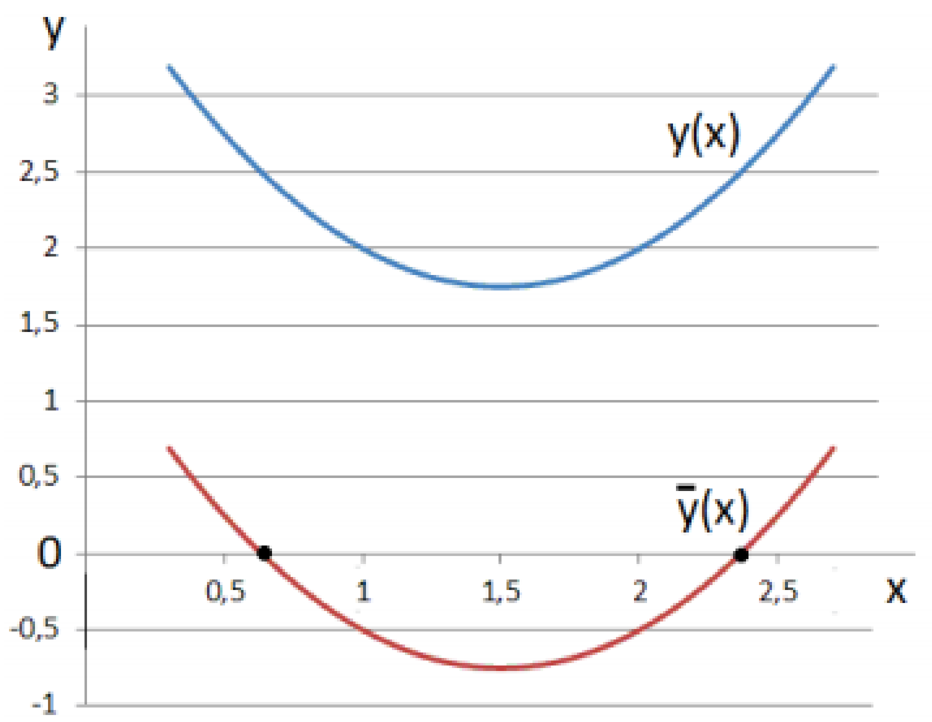
Keywords: Complex numbers, Imaginary unit, square equation, quadratic equation, Visualization of complex solutions, complex roots, Parabola, Saddle-Shaped Surface.

INTRODUCTION:

The concept of a complex number. This concept is introduced in secondary school. It is possible to review a huge number of textbooks, teaching aids [1-5], where examples of solving square equations with "no solutions" are given, or more precisely, there are two solutions, but they are imaginary, so it is impossible to show the square function of these solutions on the graph. The graph of a square function $y(x)$ is drawn in plane OXY, and a complex plane with a real axis x and an imaginary axis Im is drawn separately, where the points imitating these imaginary solutions are shown. A student experiences a state of stupor (shock) - here is the graph, there are no these solutions on the graph, and they are told that the solutions exist, but they are in another place, in some complex plane, which has an imaginary axis.

It is necessary to explain in such a way that the student sees where these solutions are on the curved line (graph). The graph is there, the solutions are there (they are calculated), but where are those solutions on the graph? This also refers to a student studying subjects that use transformation formulas with complex numbers, such as the basics of automatic control systems.

Consider a square function $y(x)=x^2+ax+b$. Let $b=-3$, $a=1{,}5$, which is given $\overline{y}(x)=x^2-3x+1{,}5$ in another example $b=-3$, $a=4$ which is given $y(x)=x^2-3x+4$. Let's draw graphs (Picture 1)



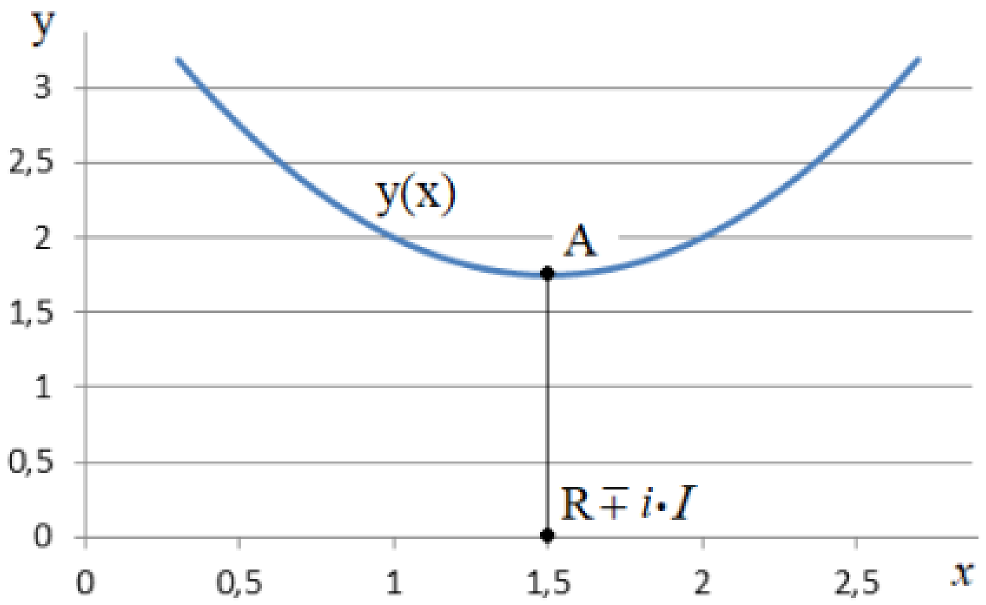
Picture 1 – graphs of square functions

If we write $y(x)=0$, then it is a square equation. We calculate solutions of the equations. For the function $\bar{y}(x)$: $x_1=\frac{3+\sqrt{3}}{2}$ and $x_2=\frac{3-\sqrt{3}}{2}$. These solutions are shown on the graph (the intersection points of the curve line with the axis x).

And for the function $y(x)$: $x_1=\frac{3+\sqrt{7}i}{2}$ and $x_2=\frac{3-\sqrt{7}i}{2}$; where is $i=\sqrt{-1}$. Where is these points of $y(x)$ on the graph? After all, there are solutions, these are x_1 and x_2, and they must be connected to the axis x (they are also denoted as x_1 and x_2).

Let's consider a square function whose "solution points are not visible" on the graph. Let function $y(x)=x^2-3x+4$ is Picture 2

Solutions of equations: $x_{1,2}=R\pm i\cdot I$, where $R=1{,}5$ and $I=\pm\frac{\sqrt{7}}{2}$. Such numbers are called complex (double) - one part is a real number R and it can be shown on the numerical axis x, the second part $i\cdot I$ is an addition to the first part. Note that the solutions differ only in the sign of the part being added. Such complex numbers are called conjugate.

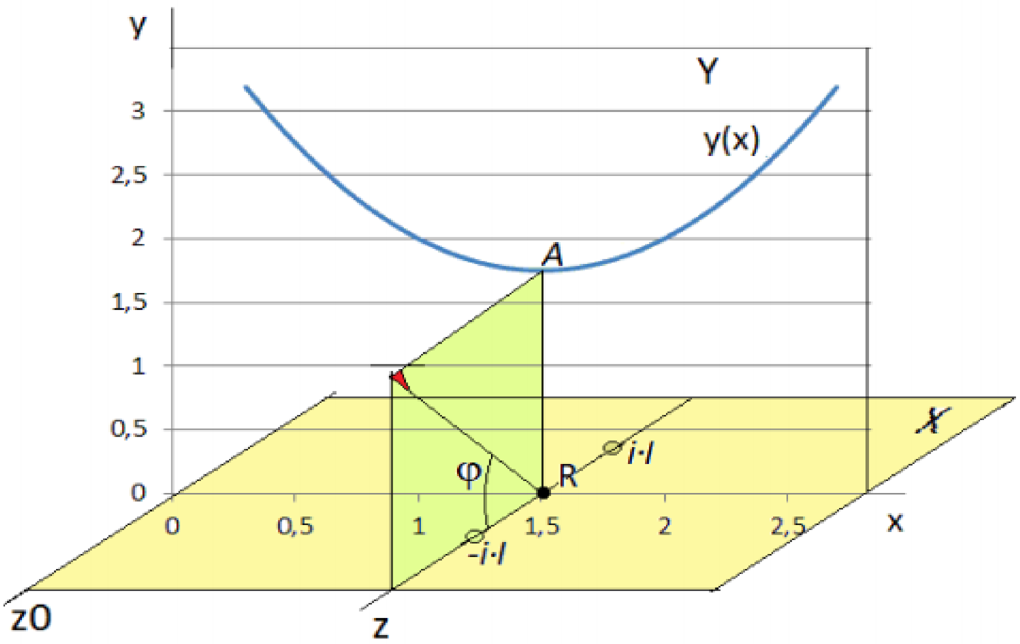


(Picture 2). A square functions with complex solutions

An axis, by definition, is the intersection of two planes: Let vertical and horizontal planes Y and X intersect, their intersection gives the axis x (Picture 3).

The graph of the function $y(x)$ is visible on the Y plane. The point R is shown on the real axis x, and and points with numbers $\pm i \cdot I$ on the axis x cannot be shown, because there are no the number $i = \left|\sqrt{-1}\right|$ on this axis. However, the numbers $\pm i \cdot I$ must be somehow connected to the axis x.

Let's look at the number axis x with the end of the axis z_0, or with the end of z. We see exactly the same as before. Therefore, it is necessary to change the point of view (to observe) to the axis x. Specifically, the viewing point of the x axis must be changed. Let's look at the axis x in the area of point R with a height above the line z. The larger the angle φ, the more clearly the points (additions) $\pm i \cdot I$ are revealed. (Picture 3).



Picture 3 – a square function

Such an idea was noted in [6], that to detail (visualize) the invisible characteristic of an object, it is necessary to change the direction of observation of the object in such a way as to increase the dimensionality of the observed space by one.

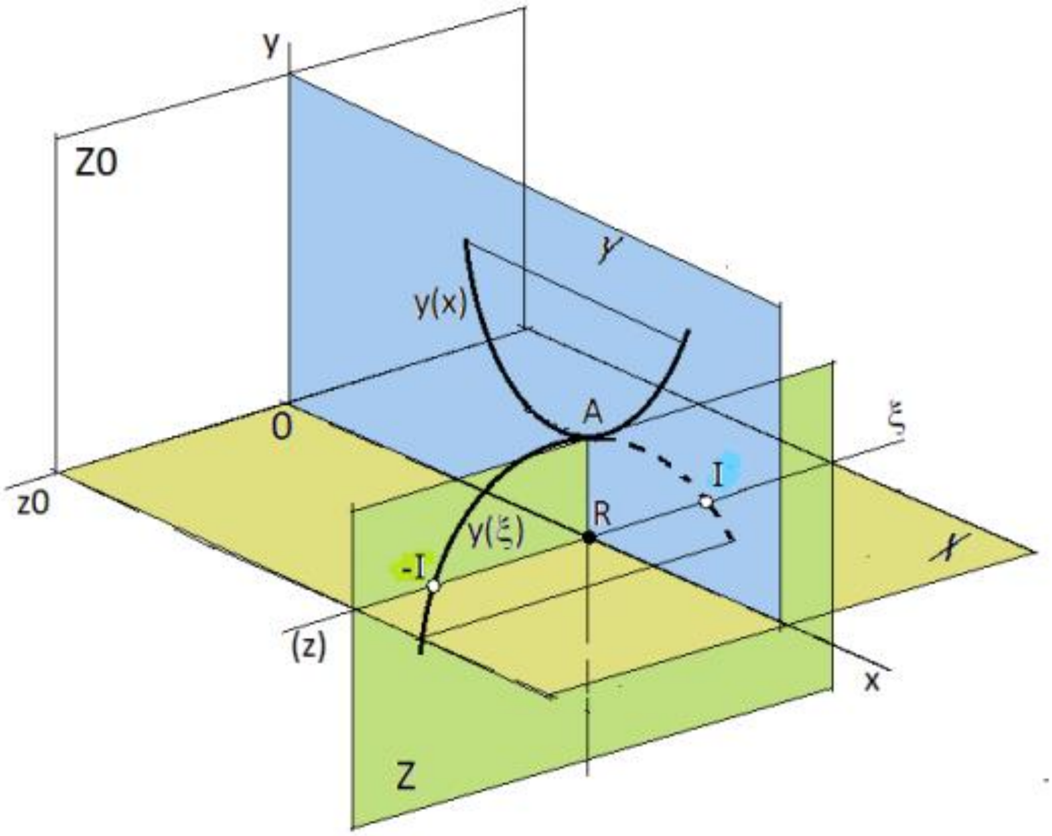
So (Picture 3), the solutions of equation $x_{1,2} = R \pm i \cdot I$ were found in the plane X, that is, the number R on the x axis, the addition $\pm i \cdot I$ on the z axis passing through point R. The numbers x_1 and x_2 are the solutions to equation $y(x) = x^2 - 3x + 4$. The graph of function $y(x)$ is drawn on the vertical plane Y.

In the vertical plane Z perpendicular to the planes X and Y (Picture 5), we construct the graph of the function

$$y(\xi) = -\xi^2 - b\xi + a;$$

$$a = y(x)\big|_{\min} = AR = 1{,}75; \quad b = 0.$$

Here, the function $y(\xi)$ is a parabola directed downward (negative sign) and its vertex is tangent to point A, for this reason $a = 1{,}75$. The coefficient b characterizes the shift of the parabola in the direction of the axis ξ. The origin of the ξ axis is taken at point R, therefore $b = 0$ (Picture 5). Thus, the equation $y(\xi) = -\xi^2 + 1{,}75 = 0$ has solutions $\xi_{1,2} = \pm I = \pm 1{,}323$.



Picture 5 – solutions of a square equation

It can be seen that points $\pm I$ are located on the axis ξ and the graph of function $y(\xi)$ passes through the horizontal plane X. The functions $y(x)$ and $y(\xi)$ are tangent at point A and are located in mutually perpendicular planes. These functions belong to the same saddle-shaped surface (hyperbolic paraboloid) and represent the trace (section) of the surface on the specified mutually perpendicular planes Y and Z.

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