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Many Participatory One Differential In The Game Persecution To Pursue

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Abstract: This at work R^k we look at the differential game in space. Our goal chaser and fugitive between in relationships Local run away to leave issue solution enough condition seeing This problem is convex. compact edge at points to normal has hyperbolic U set with $v_1$from other general to the point has not to be This lemma is the converse. hypothesis way with proven.

Keywords: Differential game, compact set, hyperplane, normal, chaser, escaper, measurable function, chase issue, local escape to leave issue.

INTRODUCTION:

Differential equations differential to the games take incoming is considered a major science. Since the 19th century in this subject textbooks create started. Differential equations fundamentals with analytical that is mainly formulas with work seer is science. From this outside geometric in the language written many We also encounter information possible. In this study, "Many participatory one differential in the game persecution to pursue. about the” lemma and theorems seeing we went out and reverse hypothesis roads with We have proved that the differential games theory dynamic systems opposite to goals has was participants between management processes research provider current from directions is one. This theory within persecution and escape issues separately place They are real processes, in particular, military strategy, aviation management, robotics, security systems and intellectual agents under control wide is used. Therefore, many participatory dynamic in environments decision acceptance to do processes mathematician modeling and analysis to do modern management theory the most complicated and important from directions one is considered. Many with agent persecution in games chaser and fugitive objects movement differential equations with representation, management limited, spatial of the sectors compactness, as well as hyperplanes with limited optimal strategies in regions determination of issues complexity In these processes, normal vectors, dimensional functions, optimal management, and strategic stability concepts central importance profession Especially one the fugitive one how many pursuers by persecution to grow in the scenario controls between coordination, resources distribution and optimal lines determination issues deep theoretical analysis demand does. Pursuit process success there is in space pursuers by effective strategies choice and fugitive local or global escape to leave opportunities limiting the conditions determination with directly It depends. Therefore for, such systems for enough and necessary the conditions to find, that is of persecution done increase or fugitive escaped of departure prevent to take according to mathematician criteria working output, theoretical and practical in terms of big importance has. This research many participant one the fugitive differential in the game persecution to grow circumstances analysis to make the optimal joint action of the pursuers strategy justification , as well as persecution done increase provider mathematician the conditions to determine Research results many with agent management in systems effective strategies working exit and complicated dynamic in environments security and stability in providing significant scientific and practical conclusions to give is expected.

The matter to be put.

R^k $(k \geq 2)$ in space $m+n$Let's consider a differential game with t players. In this game, $n$there are t pursuersP_1, . . . , P_n and mThe escapees $E_1, \dots, E_m$are participating. The actions of the pursuers are correspondingly

$$\dot{x}_i = u_i, \ \ u_i \in U, \ \ i = 1, \dots, n$$

equations through The refugees movement suitable accordingly

$$\dot{y}_i = u_i, \ \ u_i \in U, \ \ i = 1, \dots, n$$

equations through is expressed.

$t = 0$ The initial states of the forces $x_1^0, \dots, x_n^0$and the initial states of the escapees $y_1^0, \dots, y_m^0$are given in, where $x_i^0 \neq y_j^0 \ \ (i = 1, \dots, n; \ \ j = 1, \dots, m)$. Here is $x_i, y_j, u_i, v_j \in R^k, \ \ U \subset R^k$a convex compact set.$n$ We denote the game with $\Gamma(n, m, z^0)$one pursuer and $m$one escaper, and $z^0 = (x_1^0, \dots, x_n^0, y_1^0, \dots, y_m^0\ )$with the initial state of the players, by.

Local run away to leave issue solution enough conditions

Lemma 1. Hypothesis let's do it $v_1 - U$an edge point of a convex compact such that,v_1 from the point transient and v_1a hyperplane with a normal has no common point $U$with the set $v_1$except; $H - v_1$a hyperplane with a normal such that

1) $v_1 \in H^+$;

2) all $i = \overline{1, n}$for $x_i^0 \in H^-$;

3) some $j = \overline{1, m}$for the $y_j^0 \in H^+$,

this on the ground H^+, H^-Closed semi-spaces $H$are defined from the hypertext. In this case, $\Gamma(n, m, z^0)$the problem of escape in the game is solved.

Proof. E_j We show that the fugitive cannot be caught.$v_j(t) = v_1$ that we get. To prove the lemma reverse hypothesis let's go, that is so There is ia number and a moment of time $\tau$for which $x_i(\tau) = y_j(\tau)$the equality is true.

$$y_j(t) = y_j^0 + v_1(t),$$

$$x_i(t) = x_i^0 + \int_0^t u_i(r)dr$$

to equalities according to

$$0 = x_i(\tau) - y_j(\tau) = x_i^0 - y_j^0 - v_1\tau + \int_0^\tau u_i(r)dr.$$

From this

$$0 \leq (x_i^0 - y_j^0, v_1) - (v_1, v_1)\tau + \int_0^\tau (v_1, v_1)dr = (x_i^0 - y_j^0, v_1) \leq 0.$$

Last inequality to conditions 1)-3) of the lemma according to appropriate. So $(x_i^0 - y_j^0, v_1) = 0$and all$t$ in the $u_i(t) = v_1$. In that case

$$x_i(t) = x_i^0 + v_1 t$$

and

$$x_i(t) - y_j(t) = x_i^0 - y_j^0 \neq 0.$$

The lemma has been proven.

Lemma 2. Hypothesis let's do it $\Gamma(n, m, z^0)$There are hyperplanes p, qrelated to the game that H, Ghave a normal corresponding to , where $p, q-$ are $U$the edge points of the compact, p(respectively) $q$with pa hyperplane set having a normal Ucorresponding to (corresponding to). accordingly q) have no common point other than $(p,q) < 0$; and such that $I \subset \{1,\ldots,n\}, J \subset \{1,\ldots,m\}$there exist sets such that the following conditions are satisfied:

1) $p \in H^+, q \in G^+$;

2) $x_i^0 \in G^- \cap H^+, i \in I; x_i^0 \in H^-, i \notin I$;

3) $y_j^0 \in (H^+ \cap G^+)\backslash G,\ j \in J$;

4) $|J| \geq |I| + 1$,

this on the ground $H^+,\ H^-, G^+, G^-$Closed hemispheres $H, G$are defined from hypertexts. In that case, $\Gamma(n,m,z^0)$it is possible to avoid meeting in the game.

Proof. 3) from the condition $y_j^0 \notin G,\ j \in J$and according to this, the projections $y_j^0, j \notin J$of the points Hon the hyperplane can be considered to be different from each other. Let us introduce the notations:

$$J_0 = \left\{j |\ \rho\left(y_j^0, H\right) = \min_{k\in J} \rho\left(y_k^0, H\right)\right\},$$

y_j^0 $(j \in J_0)$ Let's say the hyperplane parallel to H_0passing Hthrough,$H_0(t) = H_0 + tp,\ l_j(t) = y_j^0 + tq, j \notin J_0$; $\tau_j \min\{\tau |\ l_j(\tau) \in H_0(\tau)\}$.

Of the fugitives strategies as follows Let's define:$E_j, j \notin J$ players strategies Optionally, $E_j, j \in J$the players' $V_j$strategies look like this:

$$v_j(t) = p, j \in J_0;$$

$$v_j(t) = \begin{cases} q, & t \in [0,\tau_j) \\ p, & t \in [\tau_j, \infty) \end{cases}, j \notin J_0.$$

$V_1, \ldots, V_m$ strategies support run away to leave possible In fact, $P_i, i \notin I$the pursuers cannot catch any of the escapees $P_i,\ i \in I$by Lemma 1 .$E_j, j \in J$ Each of $E_j,\ j \in J$,the pursuers has more than one the fugitive catch not to be able to We will show you . Conversely appropriate let it be, that is so $i \in I,\ j_1, j_2 \in J,\ t_1 > 0,\ t_2 > 0$ are found, $x_i(t)$for trajectories

$$x_i(t_1) = y_{j_1}(t_1),\ \ x_i(t_2) = y_{j_2}(t_2) \qquad (2)$$

equalities will be executed. In that case t_1At the moment $P_i$the pursuer and $E_{j_2}$the escaper $H_0(t_1)$lie on the hyperplane, and at the same time. $\rho\left(E_{j_2}, P_i\right) > 0$escapes $P_i,\ i \in I$by Lemma 1, which contradicts (2). Thus, E_{j_2}each of the pursuers $E_j,\ j \in J$,cannot catch more than one escaper and, by condition 4), $\Gamma(n,m,z^0)$it is possible to avoid meeting in the game. The lemma is proved.

Theorem. Hypothesis. let's do it $\Gamma(n,m,z^0)$in the game$q$ to normal has so There are $H_1, H_3, \ldots, H_{2l-1}$hyperplanes and $p$hyperplanes $p, q$with normal such that (where $H_2, H_4, \ldots, H_{2l}$is $U$the boundary point of the compact), $p$the hyperplane with normal Upassing through the point p(respectively q) has no common point $q$with the compact $p$except $(p,q) < 0$(respectively), $q$and $I_1, \ldots, I_l,\ J_1, \ldots, J_l$there are sets for which the following conditions are satisfied:

1) $H_j^+ \subset H_{j-2}^+,\ j = 3, \ldots, 2l$;

2) $I_s \subset \{1,2,\ldots,n\},\ J_r \subset \{1,\ldots,m\};\ \ s, r = 1, \ldots, l;\ I_s \cap I_r = \emptyset, s \neq r;\ \ J_s \cap J_r = \emptyset,\ \ s \neq r$;

3) $x_i^0 \in \overline{H_1^-},\ \ i \notin I_1 \cup \ldots \cup I_l$;

4) $x_i^0 \in H_{2r-1}^+ \cap \overline{H_{2r}^+} \cap \overline{H_{2r+1}^-} \cap \overline{H_{2r+2}^-}, i \in I_r, r = 1, \ldots, l-2;\ x_i^0 \in \overline{H_{2l-2}^+} \cap \overline{H_{2l-1}^-} \cap H_{2l}^-,\ \ i \in I_{l-1};\ \ x_i^0 \in \overline{H_{2l}^+},\ \ i \in I_l$;

5) $y_j^0 \in H_1^+ \cap H_2^-,\ j \in J_1;\ \ y_j^0 \in H_{2r-2}^+ \cap H_{2r-1}^+ \cap H_{2r}^-, r = 2, \ldots, l$,

this on the ground $H_1^+,\ H_1^-, H_2^+,\ H_2^-, \ldots, H_{2l}^+,\ H_{2l}^-$open hemispheres $H_1, H_2, \ldots, H_{2l}$were identified from hyperthesics;

6) $|J_1| + [|J_2| - |I_1|]^+ + \ \ldots + [|J_l| - (|I_1| + |I_2| + \ \ldots + |I_{l-1}|)]^+ > |I_1| + |I_2| + \ \ldots + |I_l|$, here $a^+ = \max\{a, 0\}$.

In that case $\Gamma(n,m,z^0)$It is possible to avoid meeting in the game.

Proof. Markings let's enter:

$$I_0 = I_1 \cup I_2 \cup \ldots \cup I_l, \ \ J_0 = J_1 \cup J_2 \cup \ldots \cup J_l.$$

Condition 5) of the theorem according to, $y_j, j \in J_0; x_i^0, i \in I_0$points$H_1$ and H_2Their projections on hyperplanes can be considered to be mutually different. Suppose that pa vector $H_2^-$is directed into a half-space and qa vector $H_1^+$is directed into a half-space,$y_j^0, j \in J_q$ from points Let $H_{2q-1}$the distance $d_j^q$to the hyperplanes be , $d_q = \min_{j \in J_q} d_j^q . y_j^0, j \in J_q$ from points Let $H_{2q}$the distance to the hyperplanes be $\bar{d}_j^q$, $d_q = \min_{j \in J_q} \bar{d}_j^q$.

$$H(t) = H_1 + d_1 + qt, \ \ l_j(t) = H_j^0 + pt, \ \ t \geq 0, \ \ j \in J_q, \ \ q \geq 2,$$

$$\tau_j = \min\{\tau | \ l_j(\tau) \in H(\tau)\}$$

that we will get .$E_j, j \notin J_0$ We select the strategies of the escapees as follows:

$$v_j(t) = \begin{cases} p, & t \in \left[0, \dfrac{d_j^1 - d_1}{\|q\| - \|p\| \cdot \cos(\widehat{p,q})}\right) \\ q, & t \in \left[\dfrac{d_j^1 - d_1}{\|q\| - \|p\| \cdot \cos(\widehat{p,q})}, \infty\right) \end{cases}, \ \ j \in J_1,$$

$$v_j(t) = \begin{cases} q, & t \in \left[0, \dfrac{\bar{d}_j^1 - \bar{d}_1}{\|q\| - \|p\| \cdot \cos(\widehat{p,q})}\right) \\ p, & t \in \left[\dfrac{\bar{d}_j^1 - \bar{d}_1}{\|q\| - \|p\| \cdot \cos(\widehat{p,q})}, \tau_j\right) \\ q, & t \in [\tau_j, \infty) \end{cases}, \ \ j \in J_q, \ \ q \neq 1.$$

$E_j, \ j \in J_q, \ q \geq 2$ The movement of the escapee is as follows: $\left[0, \frac{\bar{d}_j^1 - \bar{d}_1}{\|q\| - \|p\| \cdot \cos(\widehat{p,q})}\right)$in the interval the escapees $H_2$are placed on a hyperplane parallel to, $\left[\frac{\bar{d}_j^1 - \bar{d}_1}{\|q\| - \|p\| \cdot \cos(\widehat{p,q})}, \tau_j\right)$in the interval the escapees $H(t)$move towards the hyperplane, $E_j$the escapee $\tau_j$at the moment arrives at this hyperplane and remains there.

According to Lemma 1 optional $x_i(t), \ i \notin I_0$ trajectory for The relation $x_i(t) \neq y_j(t), t \geq 0, j \in J_1$holds. From this $x_i(t) \neq y_j(t), t \geq 0, j \in J_0$follows.

If $i \in I_p$, then we show that $P_i$the pursuer $l - p + 1$cannot catch more than ta escapees. From Lemma 1, it follows that $x_i(t) \neq y_j(t), j \in J_r, \ r \leq p, \ t \in [0, \tau_j)$. Moreover, $j \in J_r, \ r \leq p$for all $y_j(\tau_j) \in H(\tau_j)$and $j \in J_s$, $s > p$for all $y_j\left(\frac{\bar{d}_j^s - \bar{d}_s}{\|q\| - \|p\| \cdot \cos(\widehat{p,q})}\right) \in \widetilde{H}_s$, the relations hold, where $\widetilde{H}_s$the hyperplane is $\widetilde{H}_s \parallel H_2$. If $x_i(t^*) \in H(t^*)$, then $P_i$the pursuer (t^*, ∞)cannot catch any of the escapees Tin the interval. If $E_j, \ j \in J_0$up to the moment $x_i(t) \notin H(t)$, then $P_i$the pursuer cannot catch more than one of the escapees $[0, T]$in the interval $E_j, \ j \in J_s, \ s > p$. It is clear from this that $P_i$the pursuer $l - p + 1$cannot catch more than ta escapees.

Theorem proof continue We will mark. Let us introduce: $r_p = [|J_p| - (|I_1| + |I_2| + \ \ldots + |I_{p-1}|)]^+, p = 2, \ldots, l$. If all $p = 2, \ldots, p, r_p = 0$then the proof of the theorem follows from Lemma 2. Assumption let's do it all $p = 2, \ldots, l$ Let s be for $r_p > 0$s . According to the above considerations, $P_i, \ i \in I_p$each of the pursuers $l - p + 1$cannot catch more than t escapees. From this, $P_i, \ i \in I_0$all of the pursuers$l|I_1| + (l-1)|I_2| + \ \ldots + 2|I_{l-1}| + |I_l|$ from the body more than the fugitive catch can't. In that case

$$|J_1| + |J_2| + \ \ldots + |J_l| - [|I_1| + (l-1)|I_2| + \ \ldots + 2|I_{l-1}| + |I_l|]$$
$$= |J_1| + r_1 + r_2 + \ \ldots + r_l - (|I_1| + |I_2| + \ \ldots + |I_l|) > 0.$$

From this under consideration in case from meeting deviation possibility come it comes out.

Now all We are looking at the situation in some $p$countries $r_p > 0$, not in others $p.p$ Let us define the following values $p_1, p_2, \ldots, p_q$of : ; $1 \leq p_1 < p_2 < \ \ldots < p_q \leq l$. Below $J_1', \ldots, J_{q+1}', I_1', \ldots, I_{q+1}'$we introduce the sets:

$$J_1' = J_1, \qquad J_{s+1}' = J_s, \qquad s = 1, 2, \ldots, q,$$
$$I_1' = I_1 \cup I_2 \cup \ldots \cup I_{p_1 - 1}, \qquad I_2' = I_{p_1} \cup I_{p_1+1} \cup \ldots \cup I_{p_2-1},$$
$$I_s' = I_{p_{s-1}} \cup I_{p_{s-1}+1} \cup \ldots \cup I_{p_s - 1}, \qquad s = 3, \ldots, q.$$

In that case For $J_1', \ldots, J_{q+1}', I_1', \ldots, I_{q+1}'$sets and $H_1', \ldots, H_{2q+2}'$hyperplanes, all the conditions of the theorem are satisfied, including . $r'_s = [|J'_s| - (|I'_1| + \ldots + |I'_{s-1}|)]^+ > 0,\ s = 2, \ldots, q+1$ Here. $H'_1 = H_1,\ H'_2 = H_2, \ldots,$ $H'_{2s+2} = H_{2p_s-1},\ H'_{2s+2} = H_{2p_s}$This $\Gamma(n, m, z^0)$shows that it is possible to avoid meeting in the game. The theorem is proved.

CONCLUSION

Received results many with agent management in systems strategic mutual effects modeling and to them based algorithms working exit for theoretical basis become service does. The pursuit guaranteed done to increase related criteria to be determined and robotics, security systems, military management and intelligent transportation systems such as practical in the fields high to efficiency achieve opportunity The research scientific results differential games theory further development, as well as complex dynamic optimal control in processes strategies create according to next research for important methodological basis creates.

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