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Mathematical Induction In Proving Theorems

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Abstract: This article presents the proof of a number of theorems using the convenient method of mathematical induction.

Keywords: Mathematical induction, proper fractions, canonical representation of a number, positional systematic number.

INTRODUCTION:

Proving problems, as well as proving theorems, properties and propositions, and memorizing them, creates a number of difficulties for students. In many cases, for this reason, problems that require proof are neglected by students. Such problems can often be overcome by using the method of mathematical induction. That is, some theorems, identities, propositions and properties can be proved more easily by the method of mathematical induction than by other methods of proof. Below we present the proofs of several such theorems using the method of mathematical induction.

Theorem 1.

$$A_1 \times A_2 \times \ldots \times A_s = \{< x_1, x_2, \ldots, x_s > x_1 A_1 \in x_2 A_2 \in \ldots \in x_s A_s\}$$

Let $N(A) - A$ denote the number of elements of the set.

$$N(A_1) = k_1$$

$$N(A_2) = k_2$$

$$-------$$

$$N(A_s) = k_s$$

Then we have $$N(A_1 \times A_2 \times \ldots \times A_s) = k_1 \cdot k_2 \cdot \ldots \cdot k_s$$

We shall use the method of mathematical induction to prove this theorem.

Proof. Here $s \geq 2$

$$s = 2, \quad N(A_1) = k_1, \quad N(A_2) = k_2$$

$$\begin{matrix} 1 & k_2 \\ 2 & k_2 \\ 3 & k_2 \\ \vdots & \vdots \\ k_1 & k_2 \end{matrix} \qquad \begin{matrix} 1 & 1 & 2 & 3 & \dots & k_2 \\ 2 & 1 & 2 & 3 & \dots & k_2 \\ 3 & 1 & 2 & 3 & \dots & k_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_1 & 1 & 2 & 3 & \dots & k_2 \end{matrix} \qquad k_1 \cdot k_2$$

Thus, it follows that $N(A_1 \times A_2) = k_1 \cdot k_2$ so the statement is valid for $s \geq 2$ Let us assume that for

$$s-1; \quad N(A_1 \times A_2 \times \dots \times A_{s-1}) = k_1 \cdot k_1 \cdot \dots \cdot k_{s-1}$$

holds.

For

$$s; \quad N(A_1 \times A_2 \times \dots \times A_s) = k_1 \cdot k_2 \cdot \dots \cdot k_s$$

We shall prove that

$\forall n \in N$ canonical form of the number.

$$\begin{gathered} n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot \dots \cdot p_s^{\alpha_s} \\ d = p_1^{\beta_1} \cdot p_2^{\beta_2} \cdot \dots \cdot p_s^{\beta_s} \\ 0 \leq \beta_1 \leq \alpha_1 \\ 0 \leq \beta_2 \leq \alpha_2 \\ 0 \leq \beta_3 \leq \alpha_3 \\ ------ \\ 0 \leq \beta_s \leq \alpha_s \end{gathered} \qquad (1)$$

The number of elements in A_1 is $N(A_1) = \alpha_1 + 1$

The number of elements in A_2 is $N(A_2) = \alpha_2 + 1$

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The number of elements in A_s is $N(A_s) = \alpha_s + 1$.

Therefore

$$N(A_1 \times A_2 \times \dots \times A_s) = (\alpha_1 + 1)(\alpha_2 + 1)\dots(\alpha_s + 1)$$

Where *s* is the number of factors.

When finding the number of positive divisors of a number $n \in N$ one can make use of the above theorem.

$$\tau(n) = N(A_1 \times A_2 \times \dots \times A_s) = (\alpha_1 + 1)(\alpha_2 + 1)\dots(\alpha_s + 1) = \prod_{i=1}^{s}(\alpha_i + 1)$$

Theorem 2. If

$(a_1, a_2) = \delta_1, \quad (\delta_1, a_3) = \delta_2, \quad (\delta_2, a_4) = \delta_3 \dots, (\delta_{n-2}, a_n) = \delta_{n-1},$ therefore $(a_1, a_2, \dots a_n) = \delta_{n-1}$.

Proof. (by the method of MMI) 1) For *n=2* the theorem holds, since $(a_1, a_2) = \delta_1$.

2) Assume the theorem is true for (*n-1*) that is, suppose

$$(a_1, a_2) = \delta_1, \quad (\delta_1, a_3) = \delta_2, \dots, (\delta_{n-3}, a_{n-1}) = \delta_{n-2}$$

And hence

$$(a_1, a_2, ..., a_{n-1}) = \delta_{n-2}$$

3) We now show that the theorem is correct for $\forall n$ number. We have

$$(a_1, a_2, ..., a_{n-1}) = \delta_{n-1}$$

It is enough for us to show that $(\delta_{n-2}, a_n) = \delta_{n-1}$. This follows from the following theorem.

Theorem 3. If $(a_1, a_2, ..., a_{n-1}) = \delta_{n-1}$ and $(\delta_{n-1}, a_n) = \delta$ then $(a_1, a_2, ..., a_{n-1}, a_n) = \delta$.

Thus, Theorem 2 holds for every natural number $\forall n$. This theorem can also be proved by means of the method of mathematical induction.

Theorem 4. For $(a_1, a_2, ... a_n) = 1$ to hold, it is necessary and sufficient that there exist integers $\exists x_1, x_2, ..., x_n \in z$ such that $1 = a_1 x_1 + a_2 x_2 + ... = a_n x_n$

Proof. (by the method of MMI)

For n=2 it follows that the representation $(a_1, a_2) = 1 \Rightarrow a_1 x_1 + a_2 x_2 = 1$ is necessary and sufficient. We prove this using the following theorem.

Theorem 5. For *(a,b)=1* it is necessary and sufficient that $1 = ax + by \quad (\forall x, y \in z)$

Proof: 1) was $\tau_n = \delta = (a,b) = ax + by$. If *δ=1*, is $1 = ax + by$.

2) Let *1=ax+by* . We argue by contradiction

$(a,b) = d > 1$ (that is, they are not coprime).

In this case $a \vdots d$ and $b \vdots d \Rightarrow ax \vdots d$ and $by \vdots d \Rightarrow (ax+by) \vdots d \Rightarrow 1 \vdots d,\ d > 1$ this last statement contradicts our assumption. Our assumption is therefore false, hence *d=1* va *(a,b)=1*.

II. Assume that Theorem 4 holds for *n-1*, that is

$$(a_1, a_2, ..., a_{n-1}) = 1 \Leftrightarrow a_1 x_1 + a_2 x_2 + ... + a_{n-1} x_{n-1} = 1$$

is necessary and sufficient..

III. We now prove that Theorem 4 holds for every $\forall n \in N$

$$(a_1, a_2, ..., a_n) = ((a_1, a_2, ..., a_{n-1}) a_n) \Rightarrow$$

By Theorem 5 it follows that $a_1 x_1 + a_2 x_2 + ... + a_n x_n = 1$ and the theorem is proved.

Theorem 6. For *k≥1* for two consecutive convergents $\frac{P_k}{Q_k}; \frac{P_{k-1}}{Q_{k-1}}$ the identity $P_k \cdot Q_{k-1} - Q_k \cdot P_{k-1} = (-1)^{k-1}$ holds.

Proof. (by the method of MMI)

I. *n=1*

$$P_1 \cdot Q_0 - Q_1 \cdot P_1 = (q_1 q_0 + 1) \cdot 1 - q_1 q_0 = 1$$

$$1 = (-1)^{1-1}$$

$$1 = 1$$

II. Assume that for *n=k* the equality $P_k - Q_{k-1} - Q_n \cdot P_{k-1} = (-1)^{k-1}$ holds

III. We prove that it is also true for *n=k+1*

$$P_{k+1} \cdot Q_k - Q_{k+1} \cdot P_k = (q_{k+1} \cdot P_k + P_{k-1}) Q_k -$$

$$-(q_{k+1} \cdot Q_k + Q_{k-1}) P_k = q_{k+1} \cdot P_k \cdot Q_k + P_{k-1} \cdot Q_n - q_{k+1} P_k \cdot Q_k -$$

$$-P_k \cdot Q_{k-1} = P_{k-1} \cdot Q_k - P_k \cdot Q_{k-1} = -(P_k \cdot Q_{k-1} - Q_k \cdot P_{k-1}) = -(-1)^{k-1}$$

$$-(-1)^{k-1} = -(-1)^{k-1+1}$$

$$(-1)^{k-1} \cdot (-1) = (-1)^k$$

$$(-1)^k = (-1)^k$$

Thus the theorem $\forall n \in N$ is proved.

$q > 1, \quad M = \{0,1,2,...,q-1\}$ - are the digits.

Definition

$$a = a_s q^s + a_{s-1} q^{s-1} + ... + a_1 q + a_0 \qquad (2)$$

A number $a \in N$ written in the form $a_i \in M, \quad i = \overline{0,s}$ is called a number of the positional system with base q.

The numbers $a_0, a_1, a_2, ..., a_s$ are called the positional digits.

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