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Research Article

THE TRAJECTORY OF ORGANIZING AND TEACHING A FACULTATIVE COURSE ON NON-EUCLIDEAN GEOMETRIES AT SCHOOL

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ABSTRACT

The article reveals the trajectory of organizing and teaching a facultative course in order to familiarize students with non-Euclidean geometries in schools. It shows why students should be introduced to non-Euclidean geometries in schools, and what goals can be achieved by teaching non-Euclidean geometries. The most important aspect is the trajectory of the organization of the optional course. When organizing a facultative course, it is indicated what topics to choose, how many hours to allocate, and the program of the facultative course is developed.

KEYWORDS

Distance, metric, coordinate system, Euclidean geometry, non-Euclidean geometry, Lobachevsky geometry, Minkovsky geometry, Galilean geometry.

INTRODUCTION

Geometry is one of the fastest growing subjects. The school geometry course is also being enriched with modern innovations of geometry science. But even so, it is difficult to say that the geometry course of the school, which is enriched by these changes, is able to provide students with knowledge in accordance with the requirements of the present time.

At the time when new fields of modern science, athematics, cosmonautics relativity theory are being

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opened and improved, we believe that it is not enough to teach students only Euclidean geometry at school. In order to live in harmony with such developing scientific achievements and to acquire knowledge in accordance with these scientific achievements, to provide students with information about non-Euclidean geometries and to teach them the axiomatics of modern geometry and the existence of geometries other than Euclidean geometry in the plane, all geometries in the plane there is a need to give an understanding of.

THE MAIN FINDINGS AND RESULTS

If we pay attention to the teaching of geometry in secondary schools, we can see that knowledge about non-Euclidean geometries is given in textbooks based on brief information only in upper grades. So, in the school geometry course, students get detailed information only about Euclidean geometry, and about non-Euclidean geometries they can only get information about the history of its emergence.

In the usual teaching of geometry taught in schools (a point is the trace of a pencil on a sheet of paper, or the trace of a chalk on a blackboard, a straight line is a trace drawn by a pencil or chalk using a ruler, etc.) They are familiar with only one interpretation (imagination) of Euclidean geometry. In this case, their spatial imagination will be limited. Modern science and technology development requires comprehensive development of students' spatial imagination. It is very difficult to fulfill this requirement without getting acquainted with the elements of non-Euclidean geometries in general education schools. In particular, the introduction of the theory of relativity, which is the next achievement of mathematics, physics, astronomy, and cosmonautics, into general education schools shows the need to familiarize students with non-Euclidean geometries. For this purpose, it is appropriate to teach the elements of non-Euclidean geometries as a facultative course in general education schools [1], [2].

As a result of teaching the elements of non-Euclidean geometries, students' geometric imagination develops and the following qualities are brought up in them:

1. Students' general understanding of the axiomatic method, Euclidean geometry and non-Euclidean geometries will grow;

2. Pupils' ideas about basic geometric figures will expand;

3. Pupils' understanding of quantities, measurement and distance develops;

4. Pupils' understanding of methods of solving geometric problems will expand;

5. The relationship of geometry with other sciences and the understanding of its reality will be developed;

6. Pupils' outlook on real life and geometry develops.

Now, we will reveal the meaning of each of these qualities through examples.

1. The axiomatic method, through which getting acquainted with Euclidean geometry and non-Euclidean geometries, creates great opportunities to develop the students' geometric imagination in accordance with the requirements of the present time:

a) After getting acquainted with the axiomatic method, students' usual geometric ideas develop, because different interpretations of Euclidean geometry are shown using the system of axioms. For example, it is possible to take the interpretation of Euclidean geometry using a set of parabolic spheres. By "point" in this case, we mean all points of the usual

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Euclidean space, except for one point S. As straight lines we consider all circles passing through this point S, as well as ordinary straight lines (circles with center at infinity) passing through point S. As a plane, we consider all spheres passing through point S as well as normal planes (spheres with center at infinity) passing through point S. In this case, the new "space" created will consist of a parabolic connection of spheres with a radical center (this center is removed from space). Here, the fulfillment of all axioms of Euclidean geometry is checked (in the 11th grade geometry course) [3].

If we replace some geometric shapes with another, students will get acquainted with non-Euclidean geometries at this time. Students should understand that if we accept all axioms of Euclidean geometry except postulate V, and instead of postulate V we accept Lobachevsky's axiom of parallelism, Lobachevsky's geometry is formed. Or if we replace the concept of parallelism in Euclidean geometry with the sentence that any two straight lines intersect (in this case some axioms of groups I, II, III also change), students should understand well that Riemannian geometry is formed. Acquainting students with these geometries reveals to them the content and essence of science and technology innovations

b) An understanding of absolute geometry is given, and special features geometry is studied. It is proved to the students that the theorems of absolute geometry (perpendicularity of straight lines, equality of vertical angles, theorems about the signs of congruence of triangles, etc.) are appropriate in Lobachevsky's geometry and Galileo's geometry, and their fulfillment is checked on a pseudosphere and a sphere. When teaching Lobachevsky, Galilean and Minkovsky geometries, one should pay attention to their special features. Examples showing schematic connections of theorems, axioms and definitions clearly show students the essence of proving theorems.

c) By comparing the fulfillment or non-fulfillment of certain concepts, a precise definition of these concepts in Euclidean geometry is achieved. For example, if we consider the theorem that a circle can be drawn outside any triangle in Euclidean geometry, its proof ignores the intersection of the altitudes transferred to the middle of the sides of the triangle. However, when it was seen that in Lobachevsky's geometry there are triangles that cannot be circled outside, importance is attached to the intersection of these heights in Euclidean geometry. After that, it is shown that there are no squares, rectangles, rhombuses and trapezoids in Lobachevsky's geometry.

d) Students' ideas about geometric interpretations will expand. Various interpretations of Euclidean geometry and non-Euclidean geometry were shown.

2. Familiarity with Euclidean and non-Euclidean geometries expands students' imaginations about geometric figures. For example:

a) Students' ideas about the shortest distance expanded. Students connect two points on a pseudosphere and a sphere with the help of a regular (elastic) line, and come to the opinion that the shortest distance between two points, that is, the geodesic character of a straight line segment, depends on the plane. Then depicting figures of Lobachevsky's and Galileo's geometries on a regular blackboard does not cause misunderstanding and internal contradiction in students.

b) Students' ideas about asymptotic convergence lines will expand. In Lobachevsky's geometry, the distance between parallel straight lines decreases in the direction of parallelism. But these straight lines never

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intersect, so students' ideas about asymptotically converging straight lines are broadened.

c) Pupils'imagination about different spaces expands. Doing everything in the usual Euclidean plane limits students' spatial imagination. Due to familiarity with non-Euclidean geometries, their spatial imagination will expand and an opportunity will be opened for them to familiarize themselves with multidimensional spaces.

3. After learning about Euclidean geometry and non-Euclidean geometry, students' concepts of measuring quantities and distance will expand, that is, comparing sections in different geometries, adding and subtracting them, will develop students' concepts of measuring quantities.

4. As a result of getting acquainted with Euclidean and non-Euclidean geometries, pupils' ideas about solving geometric problems and its methods develop:

a) along with solving problems related to geometry, they also solve problems related only to Lobachevsky and Galilean geometries on the pseudosphere and sphere;

b) their ideas about solving calculation and proof problems are expanded. Along with the fact that the students will check the implementation of the calculation and proof problems of absolute geometry in Lobachevsky and Galilean geometries, they will be able to solve the problems related to Lobachevsky and Galilean geometries themselves (for example, measuring the sum of the interior angles of a triangle, a rectangle, etc.) also solve.

5. By getting acquainted with non-Euclidean geometries, students will clearly see the connection of geometry with other sciences. Currently, it is known to science that Euclidean geometry is not appropriate in real physical space. The hypothesis that Lobachevsky's geometry is valid in the macro world is being proven day by day. The meridian lines of the globe studied in astronomy and geography are Riemannian lines, or the trajectories of electrons are also good examples of Riemannian lines. Knowledge of the theory of relativity is further expanded by studying Galilean geometry.

6. Familiarity with Euclidean geometries develops students' worldview about real life and geometry. The scientists who lived and worked in the late 19th century and the first half of the 20th century believed that Euclidean geometry is the only possible geometry, and were against other geometries. The great Russian mathematician N.I. Lobachevsky attacked such views and created his geometry, which fully reflects the vital physical space. In addition, there is Galilean geometry, one of the non-Euclidean geometries, which can be considered as a mathematical model of the theory of relativity [4], [5].

Taking into account the above, we have set the topic "Elements of non-Euclidean geometries" as an optional course for students at the school.

Now, if we pay attention to the purpose of teaching facultative courses in schools, the following two purposes can be indicated:

1. Expanding and deepening the existing knowledge;

2. Teaching topics that are not included in the textbook, but which help to acquire more perfect knowledge of science through teaching, and then include these topics in the curriculum.

Providing knowledge about non-Euclidean geometries through optional courses enables the implementation of the above two objectives required of optional courses.

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Therefore, the selection of topics for teaching the elements of non-Euclidean geometries as an optional course and the development of teaching methods are one of the most important issues that bring school geometry closer to modern geometry.

When discussing the issue of teaching the elements of non-Euclidean geometries at school, many of our scientists emphasized that teaching should start from the upper grades. We do not agree with this opinion. We believe that it is enough for students to have knowledge of the 7th grade level to teach the elements of non-Euclidean geometries. So, the elements of non-Euclidean geometries can be taught starting from the 7th grade. This is due to the fact that in the 10th or 11th grade, when students think of geometry, only Euclidean geometry is embodied in their minds, and this may cause a little difficulty in accepting the new geometry. To teach non-Euclidean geometries, it is enough for students to know concepts such as point, straight line, coordinate system, parallelism and perpendicularity, distance, angle.

We follow the following guidelines for the selection of topics for the organized optional course and its organization:

1. If students are introduced to non-Euclidean geometries starting from the 7th grade, students will study both Euclidean and non-Euclidean geometries in parallel. By comparing the differences and common aspects of each geometry, knowledge is further expanded. As a result of skepticism about the concepts of non-Euclidean geometries, the desire to engage in scientific activities is formed in students. Even so, we consider it appropriate to teach this optional course in upper grade.

In this optional course, students will be introduced to Lobachevsky and Galilean geometries. These geometries are studied sequentially.

2. In order to organize the teaching of Lobachevsky and Galilean geometries, introducing various new concepts, students should not have insecurity or internal contradictions. Here, it is appropriate to use real-life examples and problems for students.

3. As each concept is learned (angle size, distance, parallelism, triangle, area, etc.), these concepts are taught in comparison with the concepts of Euclidean geometry, which motivates a deeper study of the essence of these concepts.

During the teaching of this course, students must be presented with various issues, especially real-life examples, and relevant examples should be developed.

We plan the optional course “Elements of non-Euclidean geometries” in schools for 34 hours. We will focus on the teaching trajectory of the optional course on non-Euclidean geometry at school in terms of content and methodology. Experience shows that the optional course “Fundamentals of non-Euclidean Geometry” should be implemented on the basis of the following program.

Elective subject (optional course) “Fundamentals of non-Euclidean geometry” (34 hours)

10th grade

1. Letter of explanation.

This course is aimed at deepening and expanding knowledge of geometry. For many students, the transition from plane geometry to space geometry is

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difficult. Developing spatial imagination requires a lot of time and a variety of exercises and tasks, from the simplest to the most difficult.

"Fundamentals of non-Euclidean geometry" competition subject is one of the most amazing and huge discoveries in the history of mathematics - the great Russian scientist N.I. Lobachevsky is called non-Euclidean geometry. The creation of non-Euclidean geometry not only helped open up new horizons in mathematics, but also made a huge contribution to the development of modern space theory. The formation of geometric images is an important part of intellectual education, polytechnic education, and is of great importance in all cognitive activities of a person.

The purpose of the course is to generalize the properties of geometric shapes and to study them in depth, to develop logical thinking. The course is aimed at developing ideas about the leading mathematical method of knowing reality - mathematical modeling, forming a holistic natural-mathematical component of the image of the world.

Examining the questions of the course is aimed at students' understanding of the multidimensional nature of mathematics, the organic combination of theoretical and practical aspects, which helps to establish fundamental internal connections, the opportunity to choose an independent field of activity; and preparing for research work at the intersection of different departments of mathematics.

The course is characterized by a judicious combination of logical rigor and geometric clarity. The theoretical importance of the studied material increases, the internal logical connections of the lesson expand, the role of deduction increases, and the level of abstraction of the facts under consideration increases.

Students learn analytical and synthetic methods of activity.

Objectives:

- to educate the understanding of the importance of mathematics for scientific and technical development;

- expanding students' ability to adapt to the modern world;

- formation of students' understanding of the role of mathematical knowledge as a means of seeing the diversity and uncertainty of the surrounding reality;

- mastering the basics of mathematical culture by students, forming a personality.

Main objectives:

1. To help students determine their own destiny by being in a situation of independent choice of an individual educational trajectory.

2. Activation of cognitive activity of schoolchildren.

3. Increasing students' information and communicative competence.

4. Providing pedagogical conditions for the flourishing of the student personality and his creative potential.

5. Increasing the amount of mathematical knowledge.

Solving the set tasks is an additional factor in the formation of positive motivation in learning mathematics, as well as students' understanding of the philosophical postulate about the unity of the world and the position about the universality of mathematical knowledge.

2. Content of topics.

Euclidean geometry - 10 hours.

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- System of axioms in Euclidean geometry;

- Properties of the system of axioms. Euclid's "Fundamentals";

- The fifth postulate and parallelism axiom;

- Theorems equivalent to Euclid's fifth postulate;

- Different ideas in solving the problem of the fifth postulate;

Some geometries in the affine plane - 24 hours.

Methods of measuring distance and angle. Modern definition of geometry. Lobachevsky geometry. Basic concepts of Minkowski geometry. Concept of circle and angle in Minkowski geometry. Planimetry of the Galilean plane. Circle and Angle in Galilean Geometry. Properties of triangular elements. Polygons. Basic concepts and their applications. Projective conjunction. About nine geometries in the plane. Examples and problems.

3. Requirements for the level of preparation of students.

As a result of studying the course, students will:

- concepts of axioms, postulates. According to Euclid, they can distinguish these concepts;

- model concept;

- properties of the system of axioms;

- Euclid's system of axioms;

- Gilbert's system of axioms;

- the role of axiomatics in geometry;

- Distance and angle measurement methods;

- Basic concepts of Lobachevsky geometry;

- Concept of Minkowski geometry;

- circle and angle in Minkowski geometry;

- planimetry of the Galilean plane;

- Circle and angle relations in Galilean geometry;

- being able to independently solve examples and problems;

- being able to model life issues;

- Differences and common aspects between Euclidean and Galilean geometries;

- they should understand the common aspects and differences in the system of axioms under consideration.

They are required to have the following skills:

- mastering methods of distance and angle measurement;

- to distinguish existing geometries in the affine plane;

- to be able to draw each geometric shape based on clear ideas about it;

- construction of polynomial sections based on accepted axiomatics and proven course theorems;

- proving theorems in solving problems, reasoning based on evidence;

- to be able to clearly imagine these concepts by learning concepts such as surface area and angle size based on each geometry;

- being able to model life issues based on the knowledge learned in geometry;

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- to be able to distinguish different geometries using distance and angle measurement methods;

- to be able to understand 9 geometries in the plane;

- in practical activities and modeling life situations in geometric language;

- checking the presented situations, predicting the result;

- it is necessary for them to have the opportunity to develop ideas about space in the universe.

4. Calendar - thematic planning.

Euclidean geometry - 10 hours

1. When and how did geometry appear? Euclid's attempts to create the first model of geometry - 1 hour;

2. Gilbert geometry model – 1 hour;

3. Consistency of the system of axioms - 1 hour;

4. Independence and completeness of the system of axioms - 1 hour;

5. Equivalence of axioms - 1 hour;

6. Euclid's work "Fundamentals" and its content - 1 hour;

7. The fifth postulate and axiom of parallelism - 1 hour;

8. The theorem about the sum of the angles of a triangle is equivalent to the fifth postulate - 1 hour;

9. The Pythagorean theorem and the fifth postulate - 1 hour;

10. Different opinions on solving the problem of the fifth postulate - 1 hour;

Some geometries in the affine plane - 24 hours

1. The concept of distance and its measurement methods - 1 hour;

2. The concept of an angle and its measurement methods - 1 hour;

3. Modern definition of geometry - 1 hour;

4. Basic concepts of geometries in the affine plane - 1 hour;

5 Basic concepts of Lobachevsky geometry - 1 hour;

6. Basic concepts of Minkowski geometry - 1 hour;

7. The concept of circle angle in Minkowski geometry - 1 hour;

8. Basic concepts of Galilean geometry - 1 hour;

9. Angle concept and its measurement in Galilean geometry - 1 hour;

10. Circle and its properties in Galilean geometry - 1 hour;

11. Equivalence properties of triangles in Galilean geometry - 1 hour;

12. Triangle height, median, bisector - 1 hour;

13. Signs of triangle equality - 1 hour;

14. Polygons - 1 hour;

15. Properties of interior angles of polygons - 1 hour;

16. Concept of surface - 1 hour;

17. Triangle surface - 1 hour;

18. Concept of cycle – 1 hour;

19. Features of the cycle - 1 hour;

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20. Problems on the mutual location of straight lines - 1 hour;

21. Problems related to polygons - 1 hour;

22. Circle, equidistant, oricycle issues - 1 hour;

23. Projective conjunction. Nine geometries in the plane - 1 hour;

24. Interrelationship of geometry departments - 1 hour;

CONCLUSION

As a result of the completion of the optional course based on this program, students' knowledge of modern geometry will be expanded, as a result of comparing different geometries with each other, students will have a better understanding of geometry and will be motivated to understand each concept more deeply.

REFERENCES

1. Gaybullayev N. Elements of non-Euclidean geometries at school. - Tashkent: "Teacher", 1971.
2. Artikbayev A., Khatamov I. Nine geometries in the plane. - Tashkent.: "Smart Reader". 2021.
3. Adajonov N.D., Yunusmetov R., Abdullayev T. Geometry part 1-2. - Tashkent.: "Teacher". 1996.
4. Artikbayev A. Modern view of ancient science. - Tashkent.: "Physics, mathematics and informatics" magazine, issue 3, 2004.
5. Artikbayev A., Berdiyeva O. Geometry in imagination. - Tashkent.: "Physics, mathematics and informatics" magazine, issue 3, 2011.
6. Khatamov I., Faizullayev Sh. Interpretation of the Lobachevsky plane on a hyperboloid. - Tashkent.: "Physics, mathematics and informatics" magazine, issue 1, 2019.
7. Khatamov I. Development of pupils' ideas about the elements of non-Euclidean geometry. - Tashkent.: "People's education" magazine 2020. Number 5. Page 74.
8. Khachaturyan A.V. Geometry of Galileo. - Moscow.: Library "Mathematical Education", 2005.
9. Yaglom I.M. Galileo's principle of relativity and non-Euclidean geometry. - Moscow: "Science", 1969..
10. Atanasyan L.S. Geometry of Lobachevsky. - Moscow.: Enlightenment, 2001. 335 p.
11. Gaybullaev N. Formation of geometric representations of secondary school students in the study of Euclidean and non-Euclidean geometry. – Tashkent, 1972. 39 p.