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Research Article

MODELS OF JILES-ATHERTON HYSTERESIS LOOPS AND MODELS OF MAGNETIZATION CURVES FOR MAGNETICALLY SOFT AMORPHOUS ALLOYS

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ABSTRACT

Two models of remagnetization of soft-magnetic amorphous alloys are considered: the Jiles-Atherton hysteresis model and a model of the magnetization curve. The aim of the research is to evaluate models according to the criteria of simplicity of the mathematical expressions obtained and the adequacy of the description of the magnetization phenomenon. The relative error of modeling is chosen as a criterion for the accuracy of the model. In the study, the least squares method was used to model the main magnetization curve and the method of optimizing the Jiles-Atherton hysteresis curve using experimental and reference data. It is concluded that both models of magnetization of magnetically soft amorphous alloys give approximately the same modeling accuracy.

KEYWORDS

Magnetization curve of magnetically soft amorphous material, Jiles-Atherton hysteresis model, approximating function, method error.

INTRODUCTION

The analysis of devices with ferromagnetic elements involves the approximation of the magnetization characteristics of ferromagnetic materials, for the approximation of the hysteresis loop, the Jiles-Atherton, [4, 5, 6, 7, 10] or Chan models are most often used [8, 9, 10, 11, 14, 15]. However, if the ferromagnetic core in the devices operate in saturation mode, and the hysteresis loop has an insignificant width, then in this case the main magnetization curve is used, the approximation of which is carried out using suitable mathematical expressions.

Most often, hyperbolic sine, arctangent, full and incomplete polynomials of the n - th degree are used to approximate the magnetization curve, where n is an odd integer [1, 2, 3, 17, 18]. The use of one or another method to create mathematical models of ferromagnetic devices depends on the goals set and the depth of study of the processes occurring in them.

Therefore, a comparative analysis of the description of magnetization using magnetization curves and using hysteresis loops is of scientific interest in order to identify the optimal method for a particular problem being solved, as well as an error estimate when using both methods of mathematical description of hysteresis.

METHODS

Cores made of magnetically soft amorphous steels and amorphous iron-based alloys were used as models for the study, the experimental magnetization curve of which was taken at alternating current with a frequency of 50 Hz according to the methods described in [1, 2], in particular for the AMAG 492 alloy (a close analogue of the Metalglass alloy described in [13]), for the remaining amorphous alloys magnetization data are taken from literature sources [12, 13]. The appearance of the main magnetization curves is shown in Fig. 1. It can be seen from the curves that for most soft-magnetic amorphous materials, saturation occurs already at values of the magnetic field strength , which indicates a high value of relative magnetic permeability for materials of this type.

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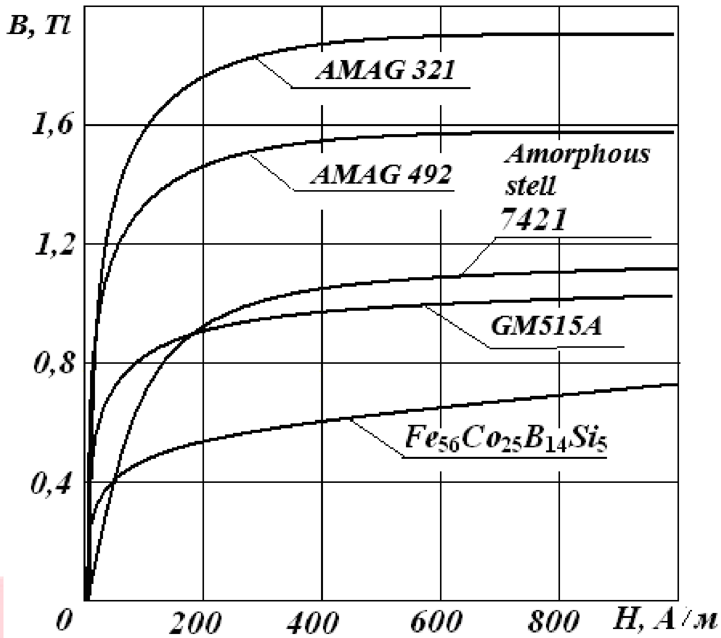

Fig. 1. Magnetization curves of common types of amorphous steels and alloys.

Linear coefficients in approximating expressions were calculated based on the minimum of the total quadratic error by the least squares method, the transition from nonlinear to linear functions was carried out using appropriate substitutions [16] and using an expression $k = \dfrac{\sum_{i=1}^{N} B_i^n \cdot \sum_{i=1}^{N} H_i - N\sum_{i=1}^{N} B_i^n \cdot H_i}{\left(\sum_{i=1}^{N} B_i^n\right)^2 - N\sum_{i=1}^{N} B_i^{2n}}$ modified for the condition of passing the curve through the origin, where N - the number of experimental points on the magnetization curve; i – the number of points; B_i, H_i, – experimental values, respectively, of magnetic induction and magnetic field strength at the -th point. For cores made of a magnetically soft amorphous iron i - based alloy of the AMG 492 brand in the range of induction variation from 0 to 1.6 Tl (saturation induction , the following approximating expressions were obtained:

hyperbolic sine $H = 1{,}892 \cdot 10^{-5} sh(11{,}552 \cdot B)$;

arctangent $B = 1{,}022 \cdot arctg(0{,}049 \cdot H)$;

an incomplete polynomial of the ninth degree $H = 14{,}66B^9$

an incomplete polynomial of the eleventh degree $H = 5{,}22B^{11}$.

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Graphs of the main magnetization curve of the amorphous AMAG 492 alloy and its approximating functions are shown in Fig. 2.

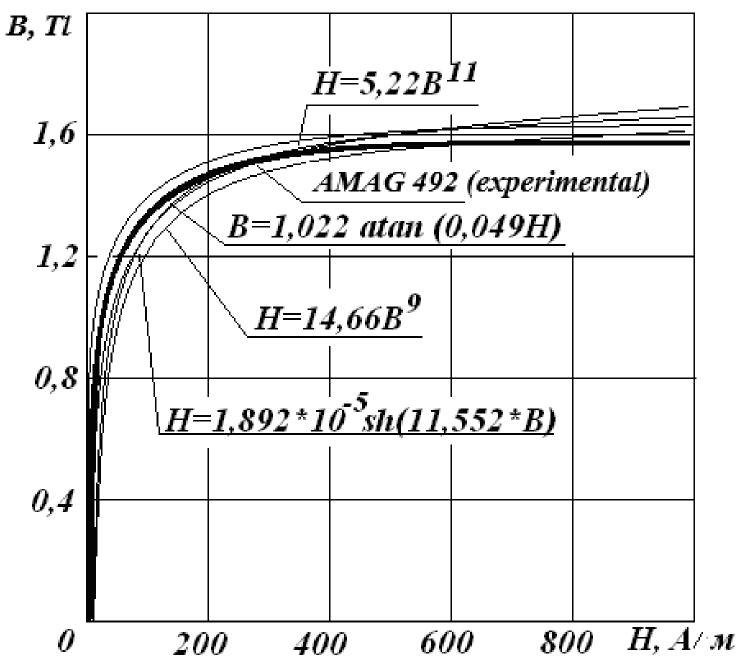

Fig. 2. Magnetization curve and its approximating functions for AMAG 492 alloy

It can be seen from the graphs of the functions that, according to the accuracy criterion, all of them are sufficiently suitable for approximating the main magnetization curve of the AMAG 492 alloy. However, expressions for hyperbolic sine and arctangent are inconvenient for subsequent transformations, in particular, expressions with hyperbolic functions are inconvenient for obtaining inverse dependencies (*H* from *B* or *B* from *H*), which is necessary when analyzing circuits. Obviously, the approximation by incomplete polynomials of the ninth and eleventh degrees is the most suitable by the criterion of simplicity and accuracy.

The relative approximation error for each of the experimental points can be calculated by the expression $\delta(\%) = \left|\frac{B_i - B_{iA}}{B_i}\right| \cdot 100\%$, where B_i - is the experimental value of magnetic induction at the *i*–th point; B_{iA} - is the value of magnetic induction calculated by the approximating function. The dependence curves for incomplete polynomials of degrees from 9 to 11, as well as for the hyperbolic sine and arctangent functions for the amorphous AMAG 492 alloy core are shown in Fig. 3.

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It can be seen from the graphs that errors in approximation by polynomials with degrees 9 and 11 give errors not exceeding 9%, which can be considered acceptable when calculating ferromagnetic elements based on amorphous alloys.

It is obvious that the methods of approximation of the magnetization curve discussed above are approximate, since in reality any ferromagnetic material is magnetized by a hysteresis loop. Therefore, a mathematical description of the magnetization process of the material, taking into account the hysteresis, is of interest. For modeling, we will use the hysteresis loop of the AMAG 492 material, using for these purposes the Jiles-Atherton hysteresis loop model [4, 5, 19], often used for modeling and calculations of ferromagnetic devices. To obtain the best accuracy of the model, it is necessary to apply its optimization, which makes it possible to calculate the optimal parameters through known experimental and reference data.

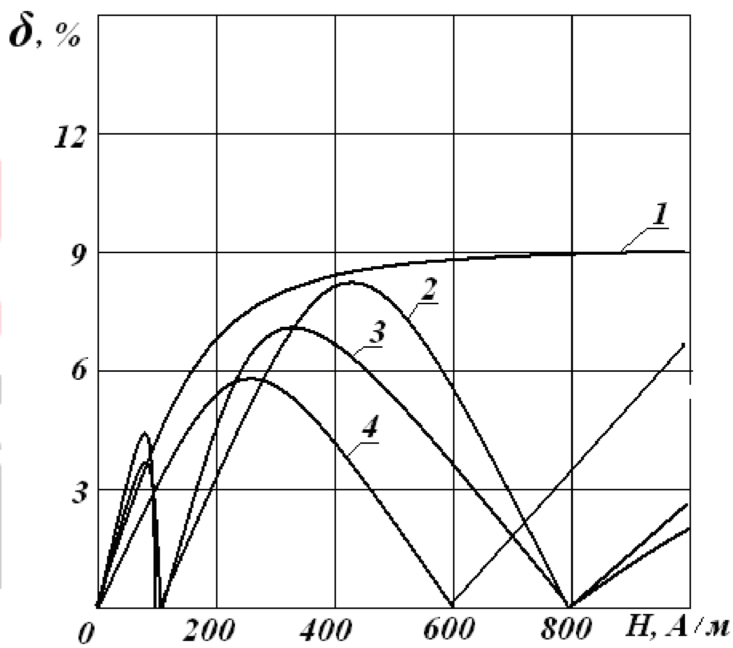

Fig. 3. Approximation errors
1–incomplete polynomial of degree 11, 2–incomplete polynomial of degree 9,
3–hyperbolic sine, 4– arctangent

The essence of the Jiles-Atherton model is that the total magnetization M consists of three components: hysteresis-free magnetization M_{an}, reversible magnetization M_{rev}, irreversible magnetization M_{irr}, and the relationship between the magnetization M, of the magnetic field strength H and the magnitude of magnetic induction B is described by the expression

$$B = \mu_0 (M + H).$$

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The magnetization M of a ferromagnet in an external magnetic field depends on the magnitude of the internal field H_i, equal to $H_i = H + \alpha M$, where α - is a coefficient that takes into account the effect of the interaction of the external and internal magnetic field. Due to the small value α equal $4-6\cdot10^{-5}$ in the sources [4], it is recommended to take it equal to zero, thus it turns ou $H_e \approx H$.

The magnitude of the hysteresis–free magnetization M_{an} can be written in the form $M_{an} = M_s f(H)$, where M_s - is the saturation magnetization, and $f(H)$ - is a function equal to zero at $H=0$ and one at H, tending to infinity. In the Jiles-Atherton model as a function $f(H)$, the Lanjevin function is used as a function in the form £(x)=coth(x)-1/x, with this in mind, the hysteresis–free magnetization curve is described by a function $M_{an} = M_s \cdot \coth\left(\left(\frac{H}{A}\right) - \left(\frac{A}{H}\right)\right)$, where A - is a scale factor ranging from 0.1 to 10000, selected by the appearance of the hysteresis loop so that the curve M_{an} passes through the points (0,0) and the (H_c, B_r) hysteresis curve, where H_c and B_r – accordingly, the coercive force and the residual magnetic induction of the investigated ferromagnetic material.

It is known from [4] that the total magnetization M is the sum of two components – irreversible magnetization M_{irr} and reversible magnetization M_{rev}

$$M = M_{irr} + M_{rev}. \qquad (1)$$

The derivatives H of the irreversible and reversible components are determined , respectively , by the expressions

$$\frac{dM_{irr}}{dH} = \frac{M_{an} - M_{irr}}{\frac{\delta k}{\mu_0} - \alpha(M_{an} - M_{irr})}; \quad \frac{dM_{rev}}{dH} = c\left(\frac{dM_{an}}{dH} - \frac{dM}{dH}\right), \qquad (2)$$

from where, after transformations and taking into account (1), a differential equation describing the hysteresis in the Jiles-Atherton model can be obtained

$$\frac{dM}{dH} = \frac{1}{(1+c)} \cdot \frac{(M_{an} - M)}{\frac{\delta k}{\mu_0} - \alpha(M_{an} - M)} + \frac{c}{(1+c)}\frac{dM_{an}}{dH}. \qquad (3)$$

Here: δ – the sign function, $\delta = 1$ if $\frac{dH}{dt} \geq 0$, $\delta = -1$ if $\frac{dH}{dt} \leq 0$, $\frac{k}{\mu_0} \approx H_c$ - a coefficient

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approximately equal to the coercive force; c – a weighting coefficient equal to the ratio of the differential susceptibilities of the initial and hysteresis–free magnetization curves, determined experimentally by the best approximation of the calculated and experimental hysteresis curves, is in the range from 0 to 1; α - a coefficient that takes into account the effect of the interaction of external and internal magnetic fields, previously its value was assumed to be zero.

Taking into account these notations, expression (3) will be rewritten as

$$\frac{dM}{dH} = \delta\frac{(1-c)\cdot(M_{an} - M)}{H_c} + c\frac{dM_{an}}{dH}. \qquad (4)$$

Integrating the left and right parts of (4) by dH , we get

$$M = \delta\frac{1-c}{H_c}\int(M_{an} - M)dH + c\cdot M_{an}. \qquad (5)$$

Since, $M_{an} = M_s\cdot\coth\left(\left(\frac{H}{A}\right)-\left(\frac{A}{H}\right)\right)$, after substituting this expression in (5), we finally get

$$M = \delta\frac{1-c}{H_c}\int\left(M_s\cdot\coth\left(\frac{H}{A}-\frac{A}{H}\right)-M\right)dH + c\cdot\left(M_s\cdot\coth\left(\frac{H}{A}-\frac{A}{H}\right)\right) \qquad (6)$$

Let us perform the integration of equation (6) by the numerical Gauss-Kronrod method - [16] as giving the highest algebraic accuracy with the following initial parameters for the AMAG 492 alloy, given below: $B_s = 0{,}75Tl$; $M_s = 1{,}27\cdot10^4\ A/m$; $H_s = 8\ A/m$; $\delta = 1,-1$; $A = 32$; $c = 0{,}58$; $a = 0$.

Based on the results of numerical integration, we obtain a number of values of the magnetic field strengths *H* and the corresponding inductions *B*, and we will take the integral within the range of the change in the magnetic field strength from *-1000* to *+1000 A/m*. Figure 4 shows graphs of the hysteresis curves of the dependence $B = f(H)$ for the AMAG 492 alloy, obtained experimentally and calculated based on the results of solving equation (6) for the steady-state mode at a magnetization reversal frequency equal to 50 Hz.

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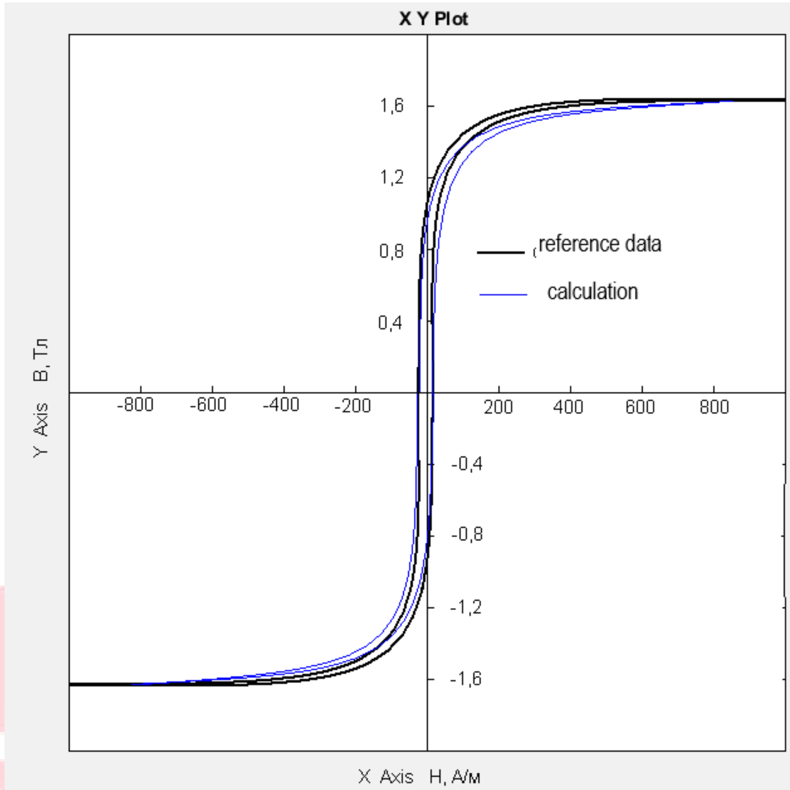

Fig. 4. Calculated and experimental graphs of hysteresis curves of dependence $B = f(H)$ for AMAG 492 alloy

From the graphs shown in Fig. 4, a good coincidence of the calculated and experimental curves can be seen, which at the reference points (the origin is exact, the point with the coercive force H_c and the residual magnetic induction B_r and the point with the limiting value of the magnetic field strength, in our case equal to 800 A/m) coincide completely. The greatest difference between experimental and calculated graphs of hysteresis loops is observed in the area of the greatest bend of the magnetization curve. In the areas of linear dependence $B = f(H)$ and the saturation area of the magnetization curve, the calculation errors are minimal.

Results

Let's compare the magnetization curves of amorphous materials obtained by approximating them with an algebraic expression and their hysteresis loops obtained using the Jiles-Atherton model. As a comparison criterion, the value of the relative modeling error calculated by the expression $\delta(\%) = \left|\frac{B_i - B_{iA}}{B_i}\right| \cdot 100\%$ can be used, where B_i - is the experimental value of magnetic induction at the i – th point; B_{iA} - is the value of magnetic induction calculated by the approximating function and using the Jiles-Atherton model at the same point.

In figure 5 shows the graphs of the dependence $B = f(H)$ for the amorphous AMAG 492 alloy, constructed for various modeling methods: the experimental dependence $B = f(H)$, taken on a full-scale sample, the calculated dependence $B = f(H)$, obtained by using an approximation by an incomplete polynomial of the form $H = 14{,}66B^9$ and a computational model of the hysteresis loop derived from the Gills-Atherton model. It can be seen from the graphs that the accepted methods give approximately the same modeling accuracy.

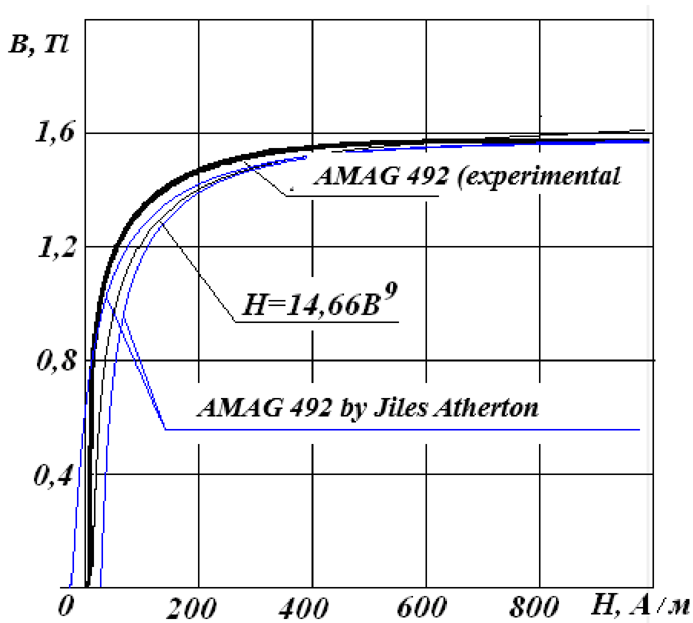

Fig. 5. Graphs of the dependence $B = f(H)$ for various modeling methods

Figure 6 shows graphs of the dependence of the relative modeling error $\delta(\%) = f(B)$ for the AMAG 492 alloy when using the modeling methods discussed above.

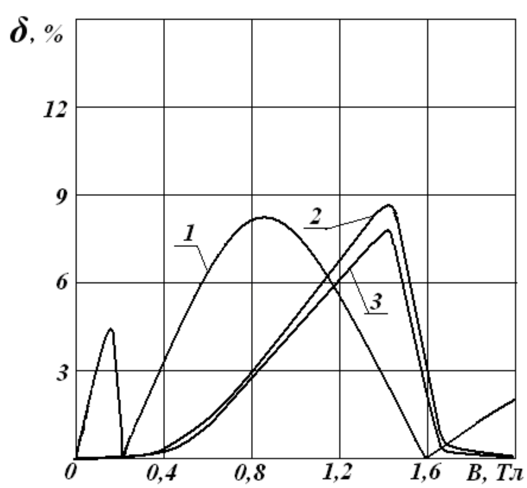

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Fig. 6. Dependence graphs $\delta(\%) = f(B)$ for AMAG 492 alloy:

1 - approximation by the function $H = 14,66B^9$, 2 – the direct branch of the hysteresis loop of the Jiles-Atherton model, 3– the reverse branch of the Jiles-Atherton hysteresis loop model

CONCLUSION

1. It is evident from the graphs shown in Fig. 6 that the relative errors of approximation of the magnetization loop and the hysteresis loop of the Jiles-Atherton model in their greatest magnitude differ little from each other, and on this basis both the model of the magnetization curve obtained by approximating the magnetization curve of a ferromagnetic material and the Jiles-Atherton model can be accepted for analysis devices based on magnetically soft amorphous materials, including those operating in saturation mode.

2. The final conclusion about the advantages of a particular model can be made only on the basis of the final goals of the analysis, since the permissible errors differ by no more than the magnitude of the measurement error and are not sufficiently reliable.

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