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Features of The Mathematical Description of Typical Links of Electromechanical Systems

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Abstract: The variety of methods for mathematical description of electromechanical sys-tems, the feasibility of combining different approaches to modeling different types of elements - all this leads to the need to take into account the specifics of the mathematical description of each element of the system. Structural and dynamic properties of typical elements of electromechanical objects are analyzed and sys-tematized.

Keywords: Electromechanical systems, mechatronic systems, modeling, actuators, mathemat-ical description.

Introduction:

Modern electromechanical systems are an integral part of technical means used in industry, mechanical engineering, energy and transport. Due to the widespread use of such objects in various areas of human activity, intensive work has been carried out for many years to improve them. Simple objects have been replaced by controlled electromechanical objects with a wide range of functions and improved performance characteristics.

Since significant energy consumption is observed during the operation of electromechanical systems, intensive work has been carried out for many years to improve their energy characteristics. The tasks of improving dynamic characteristics by using automatic control systems also remain relevant. Depending on the task, the automated control system must provide the necessary operating modes of the electromechanical system with control of speed, power, mechanical torque, frequency, etc. Currently, for modern electromechanical systems, there is a steady trend towards increasing requirements for the accuracy of complex movements of actuators, provided that their speed of movement increases. This became possible with the advent of powerful semiconductor power converters and high-precision digital control systems, which laid the foundation for the development of mechatronic systems [1-3]. Therefore, when considering the structural and dynamic features of modern electromechanical systems, it is advisable to focus on electromechanical systems of the mechatronic type. Taking into account their structural and dynamic features will help determine ways to improve the efficiency of their operation.

A feature of mechatronic systems is that they cover a wide range of tasks due to the versatility of computerized control, monitoring and diagnostic systems. Depending on the features of the actuator (control object), a mechatronic system can use a different set of sensor devices and different algorithms for generating control actions. To improve the efficiency of controlling complex multichannel and multi-connected objects, their reference and predictive models are used. Additional requirements are put forward for such models, in particular, the ability to function in real and accelerated time modes, high reliability, the ability to synchronize the current state of the model relative to a real operating object, etc. [4-6].

The use of computerized control systems in mechatronic devices has made it possible not only to improve control quality indicators, but also to solve a set of problems associated with controlling complex systems with a large number of electromechanical converters [7]. The specificity of these problems is that when generating control signals, it is necessary to take into account their interconnectedness through the commonality of the control object, which can have a complex structure. To solve such problems, it is necessary to create mathematical models of multi-connected control objects. Note that for such control objects, the construction of parametric models causes significant difficulties, so it is necessary to find other ways of their mathematical description. An effective approach to the construction of mathematical models of multi-connected dynamic objects is the use of a mathematical description in the form of integral equations and their systems using identification methods.

METHODOLOGY

A distinctive feature of modern mechatronic systems is the use of the principles of unification, aggregation and typification [1]. This allows for the design of a mechatronic system based on unified blocks, which, if necessary, can be replaced with similar standard blocks. Such flexibility of the structure of a mechatronic system allows for prompt changes in it in order to optimize the structure for a specific class of problems. In addition to structural flexibility, mechatronic systems also have software flexibility due to the use of programmable microcontrollers and microcomputers in control, monitoring and diagnostic systems. Changing control algorithms in mechatronic systems, in the vast majority of cases, is carried out by changing software modules. Another positive moment for mechatronic systems is a significant increase in the efficiency of control, monitoring and diagnostic algorithms when using mathematical models of control objects. At the same time, at the current stage of development of mechatronic systems, there is a need to create effective high-speed algorithms for modeling the dynamics of various actuators as control objects.

When describing the dynamics of complex mechanical elements (long kinematic transmissions, spatial frame structures and mechanisms), which consist of both homogeneous and heterogeneous elements (beams, rods, plates, shells, etc.) with different types of connections between themselves (connections through elastic and damping elements, rigid and movable-hinge connections, etc.), the use of a universal method of mathematical description causes significant difficulties. In addition, the presence of different types of connections and different types of motion between elements (longitudinal, transverse, torsional vibrations) also complicates the task of mathematical description. Such a variety of interaction methods can be reproduced using structural models, which can consist of different types of links combined into a single block-structural scheme.

Let us consider the dynamic properties of typical objects with distributed parameters, which are present in modern controlled electromechanical systems.

Remote rods of industrial robots. When industrial robots with long remote rods designed to carry loads operate, the elastic compliance of the robot links has a significant effect on the trajectory of the gripping unit. Due to the presence of distributed masses and elastic links, elastic oscillations occur, the amplitude of which can be unacceptably large, which causes inaccurate operation of the robot's actuators. Therefore, it is necessary to have a mathematical model of the robot links taking into account the distribution of parameters, which will allow us to study the effect of elastic deformations on the accuracy of processing program movements.

Let us consider a mechanical model of an industrial robot (Fig. 1), which consists of two rectilinear links 1 and 2 and a gripping unit [3, 8]. The first link (OO_1) is a rod of rectangular or circular cross-section and moves along the axis O_{x_3} in the vertical direction.

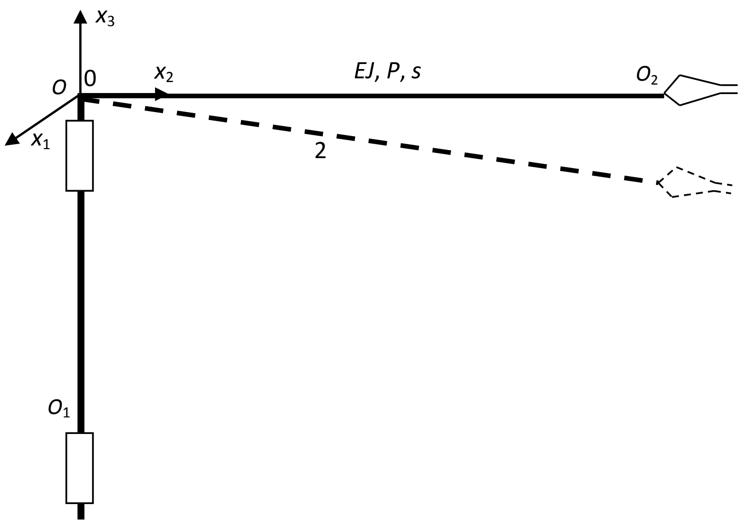

Fig. 1. Kinematic diagram of an industrial robot.

At point O there is a cylindrical hinge connecting links OO_1 and OO_2. Link 2 rotates around an axis parallel to the plane x_1x_2. At point O_2 there is a wrist hinge by means of which the gripping unit is connected to link OO_2. The moment of inertia of the load m is sufficiently small relative to the axis passing through its center of gravity perpendicular to the plane of oscillation. Then the equation of oscillation of link OO of the industrial robot has the form

$$EJ\frac{\partial^4 y}{\partial x^4}+\left[\rho s+m\delta\left(x-x_1\right)\right]\frac{\partial^2 y}{\partial t^2}=f_1\left(x,t\right)$$

where EJ, ρ, s are respectively the bending rigidity, material density and cross-sectional area of the link; y is the deflection of the x-section of link 2; f_1 is the linear density of external forces.

Introducing a system of relative units u = y/l, ξ = x/l, τ = t/t_0, f(ξ, t) = $l^3 f_1(x, t)/(EJ)$, $t_0 = l/v_0$, $v_0 = \sqrt{EJ}/(\rho s/l)$, where t_0 and v_0 are the model time and speed, we obtain the equation of motion of the second link of the industrial robot

$$\partial_\xi^4 u+\left[1+\mu\delta\left(\xi-\xi_1\right)\right]\partial_\tau^2 u=f_1\left(\xi,\tau\right)$$

with boundary conditions

$$u\Big|_{\xi=0}=\frac{\partial^2 u}{d\xi^2}\Big|_{\xi=0}=\frac{\partial^2 u}{d\xi^2}\Big|_{\xi=1}=\frac{\partial^3 u}{d\xi^3}\Big|_{\xi=0}=0$$

where μ = m/(ρls) is the ratio of the mass of the load to the mass of the OO_2 link.

Drill pipe columns of drilling rigs. When constructing wells, an important problem is to ensure high technical and economic indicators of the drilling process. This problem is especially relevant when constructing deep and super-deep wells. The main process during drilling is the operation of the bit to deepen the wellbore. A distinctive feature of automatic bit feed control systems is the presence of a drill pipe column, through which the bottomhole parameters are measured (axial load and bit movement speed), as well as the transmission of the control action from the surface to the bottomhole.

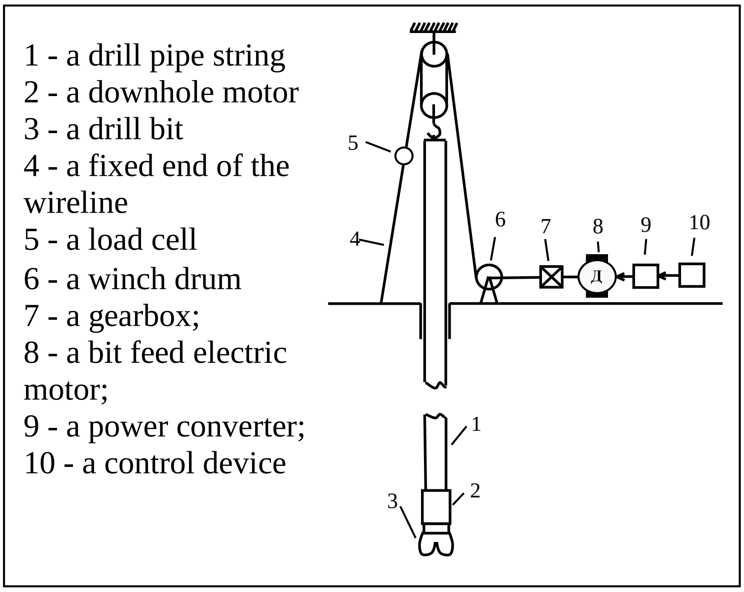

Fig. 2. Simplified diagram of the drilling rig bit feed mechanism.

The influence of the drill column is manifested in a significant distortion and delay of information received from the bottomhole, and the control action, which is transmitted in the opposite direction. Therefore, the organization of the bit feed control process is associated with significant difficulties. The productivity of drilling rigs can be increased by taking into account the dynamic characteristics of the drill string when transmitting mechanical forces from the wellhead to the bottomhole. [3, 9]

A simplified structural diagram of the drill bit feed regulator is shown in Fig. 2.

The axial reaction of the face and the reaction of the bit are applied to the lower end of the column, and the forces of gravity, viscous friction, and inertia are distributed along the length. The drill pipe column, taking into account a number of assumptions, can be considered an elastic homogeneous rod with distributed mass, elasticity, and viscous friction. The displacement of the sections of the column elements is described by a differential equation in partial derivatives

$$\frac{\partial^2 u}{\partial t^2}+2a\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}, \qquad (1)$$

where $a=h/\left(2m\right)$, $c=\sqrt{Es/m}$; m is the mass of a unit length of the column; u is the displacement of the column cross-section relative to the equilibrium position; E is the modulus of elasticity; s is the cross-sectional area of the column; x is the coordinate of the column cross-section; a is the damping coefficient; h is the damping coefficient per unit length of the column; c is the velocity of propagation of the displacement along the column.

The speed of movement of the drill pipe section v and the increase in force P are determined by the expressions $v = \frac{\partial u(x,t)}{\partial t}$, $P = -Es\frac{\partial u(x,t)}{\partial x}$.

When using a surface bit feed controller, the control action for the drill string is the displacement rate of the upper end of the string $v_0(t)$. The boundary condition for this end of the string is

$$\frac{\partial u(x,t)}{\partial t}\Big|_{x=0} = v_0(t). \qquad (2)$$

The boundary conditions for the lower end of the drill pipe string are

$$P(l,t) = -Es\frac{\partial u}{\partial x}\Big|_{x=l} = zv_l(t), \qquad (3)$$

where l is the length of the column; v_l is the bit speed; z is the parameter determining the interaction of the bit with the rock (at an absolutely hard bottomhole z → ∞). Equations (1)—(3) define the dynamic characteristics of the drill string as an object with distributed parameters. However, the presented mathematical model does not take into account the fact that real drill strings consist of parts with different physical properties and also belong to developing systems. To ensure the necessary adequacy of the mathematical model of the drill pipe column, it is necessary to involve new approaches to its construction, in particular, the structure-oriented approach, which involves representing the model of a complex dynamic object in the form of a set of mathematical descriptions that allows taking into account the features of each structural element in its mathematical description and numerical implementation. As part of the application of the structural approach to the construction and numerical implementation of models of dynamic distributed objects, it is advisable to determine, by analogy with the structural modeling of objects with lumped parameters, typical structural blocks.

RESULTS AND DISCUSSION

For this purpose, we will find out what types of actuators with distributed parameters are present in electromechanical systems. In general, they can be divided into two large groups: spatially one-dimensional and multidimensional. Spatially multidimensional ones include actuators of industrial manipulators with complex frame spatial structures, drilling rigs, etc. Spatially one-dimensional objects are divided into rod and ring-shaped. Objects of ring nature include drives of conveyors and escalators, drives for feeding manipulator platforms for reinforcement and winding, drives for positioning the heads of printing devices, etc. Rod objects are cables of lifting installations, elevators, towing systems, long kinematic transmissions, drill pipe columns, long shafts, remote rods of industrial robots. The classification scheme for objects with distributed parameters of electromechanical systems is shown in Fig. 3.

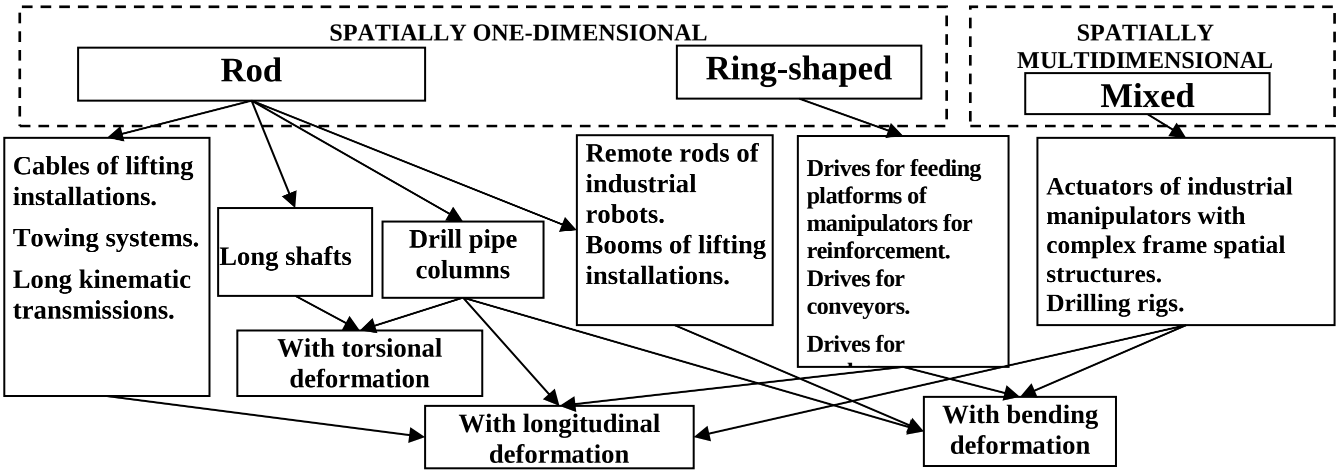

Fig. 3. Classification of actuators with distributed parameters of electromechanical systems.

Analyzing the dynamic characteristics of the objects shown in the diagram, we can conclude that when describing them mathematically, their initial models can be reduced (by decomposition) to a structure whose elements are mathematical descriptions of distributed links with three types of deformation (longitudinal, torsion and bending). Therefore, when describing the dynamic characteristics of such objects, the typical element is a linearly extended element, which, without taking into account the dissipation of energy and resistance forces, is described by a system of partial differential equations

$$\left.\begin{aligned} &E\frac{\partial^2 u(x,t)}{\partial x^2}-\rho\frac{\partial^2 u(x,t)}{\partial t^2}=0;\\ &GJ_p\frac{\partial^2 \varphi(x,t)}{\partial x^2}-I\frac{\partial^2 \varphi(x,t)}{\partial t^2}=0;\\ &EJ\frac{\partial^4 y(x,t)}{\partial x^4}-\rho s\frac{\partial^2 y(x,t)}{\partial t^2}=0, \end{aligned}\right\} \qquad (4)$$

where u(x, t) is the longitudinal shear of the section; φ (x, t) is the angle of twist of the section around the longitudinal axis; y(x, t) is the displacement of the section from the centerline in the transverse direction; G is the shear modulus; J_p is the polar moment of inertia of the cross section; I is the moment of inertia of a unit of length; ρ is the density of the substance; E is the modulus of elasticity.

CONCLUSION

Thus, the features of modern electromechanical systems should be sought in the plane of analysis of the properties of actuators (control objects) in order to obtain for them the corresponding mathematical dependencies in a form that would make it possible to effectively solve the problems of their modeling.

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