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## ON THE GENERALIZED ALGORITHM FOR OPTIMAL CONTROL OF SYSTEMS OF LINEAR DIFFERENTIAL-DIFFERENCE EQUATIONS

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**Dilshod Davronovich Aroev**

PhD, Associate Professor, Kokand State Pedagogical Institute, Uzbekistan

Orchid: - <https://orcid.org/0009-0004-0392-7597>

### ABSTRACT

This article presents a model for optimizing the number of control function parameters for objects with delay, which are expressed by a system of differential equations. It also discusses the model of the stability of the robot's motion trajectory after the practical process of a specific object, as well as algorithms for improving positional accuracy.

### KEYWORDS

Control function parameters, optimal solution, kinematic chain, system of differential-difference equations.

### INTRODUCTION

This article presents a generalized algorithm for optimizing the number of parameters of the control function in the optimal control of systems of linear differential-difference equations. The algorithm is built upon the "K-controllability" algorithm for autonomous control systems described in [1]. Starting from the second condition of the theorem provided in [2], the algorithm fully utilizes the procedure outlined in [1] (Figure 1). This includes finding the eigenvalues of the system matrix for "K-controllability" and transforming the system into Jordan normal form, as well as detailing the algorithms for achieving full controllability of the system.

The general form of a system of linear differential-difference equations is as follows:

$$\begin{cases} \dot{x}(t) = Ax(t - h) + Bu(t) \\ y(t) = Hx(t) \end{cases} \quad (1)$$

where:  $x = x(t) - n$  – a vector representing the state of the system at time  $t$ ;

$A$  – constant matrices of specific dimensions;  $(n \times n)$  – үлчовли ўзгармас матрица;

$B - (n \times m)$  – A constant matrix with non-zero elements;;

$h$  – an infinitesimally small quantity;;

$u(t) - (1 \times m)$  – a control function vector;  $h > 0, t_0 \leq t \leq T$ ;

$y(t) - n$  – a vector of specific dimensions depending on the state vector  $-(1 \times n)$ ;

$H - (r \times n)$  – A constant matrix of specified dimensions.

The application of the second theorem on optimization provided in [2] to the process of industrial robot operation indicates that the selection of  $C_k$  and  $B_k$  matrices can be represented through an  $(n \times n)$  – dimensional matrix expressed via an

$((n + 1) \times (n + 1))$  – dimensional matrix, based on the kinematic equations of the industrial robot's motion. The appearance of the  $C_k$  matrix is as follows:

$$C_k = \begin{bmatrix} C & 0 \\ 0 & 1 \end{bmatrix},$$

The appearance of the  $B_k$  matrix is as follows:

$$B_k = \begin{bmatrix} B & 0 \\ 0 & 1 \end{bmatrix}$$

To satisfy the first condition of the theorem, the  $P$  matrix is selected based on the technical and structural characteristics of the industrial robot. Then, it is verified that the equation  $PB_k = C_k P$  holds true.

$$B_k = \begin{bmatrix} B & 0 \\ 0 & 1 \end{bmatrix}, C_k = \begin{bmatrix} C & 0 \\ 0 & 1 \end{bmatrix}$$

The problem in [2], i.e., restoring damaged points using the recovery equation, similarly reduces to a system of linear equations, as in the first problem. Due to the repetitive nature of the algorithms constructed for this, we do not present the algorithms here.

1 - the beginning:

2 - initial data: the dimension of the space under consideration ( $N$ ), the initial and final time of the process:  $(t_0, T)$ , the delay time ( $h$ ), system matrices  $(A = \{a_{ij}, i, j = \overline{1, N}\}; B = \{b_{i,j}, i = \overline{1, N}, j = \overline{1, M}\} (M \leq N); P = \{p_{i,j}, i, j = \overline{1, N}\}); C = \{c_{i,l}, i = \overline{1, N}, l = \overline{1, K}\} (K \leq N)$  control function values  $u_i (i = \overline{1, N})$ , Additional matrices  $B_k = \{b_{ij}, ij = \overline{1, N}\}; C_k = \{c_{ij}, ij = \overline{1, N}\}$ , initial values of the system at  $t = t_0$ .

3 - Determining the direction of the damaged point in the system.

4 - transforming the matrix  $A$ , which represents the system's state, into its Jordan normal form  $J(A)$ .

5 - Generating the matrices  $PB_k, C_k P, BC$ .

6 - the condition  $PB_k = C_k P$  is checked.

7 - If "yes", move to block 8; if "no", move to block 2.

8 - the second condition of the theorem is checked based on the algorithm provided in [2].

9 - end.



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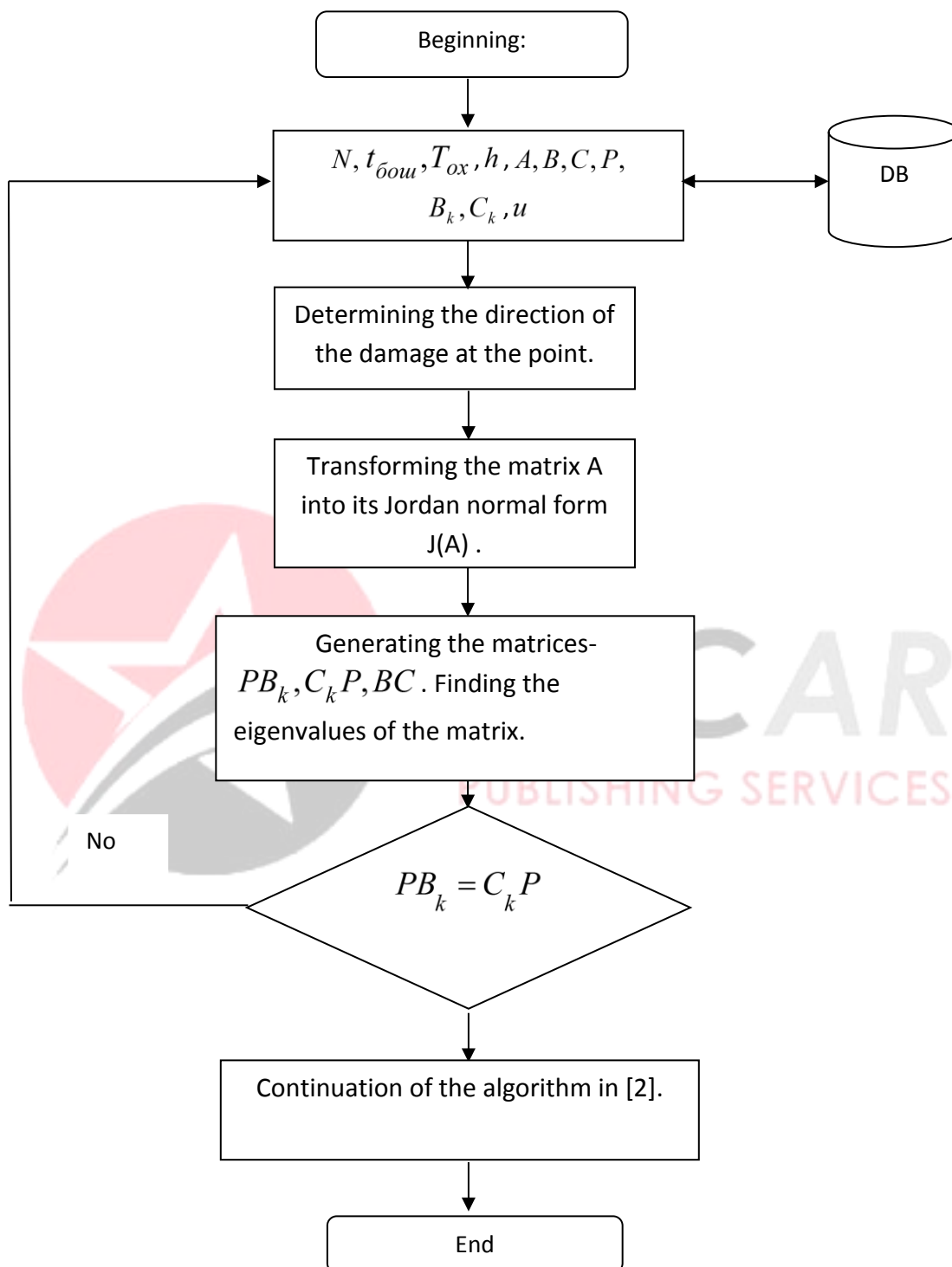


Figure 1. Block diagram of the optimization algorithm for the number of control function parameters in the control of a system of linear differential equations.



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