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Research Article

MATHEMATICAL MODEL AND NUMERICAL METHODS OF FILTRATION PROCESSES OF LIQUID SOLUTIONS

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Alisherov Akramboy Alisherovich
Samarkand City Specialized Boarding School No. 1., Uzbekistan

ABSTRACT

Filtration processes of liquid solutions are fundamental in many natural and industrial applications, such as environmental protection, chemical engineering, water purification, and petroleum extraction. This article develops a mathematical model for describing filtration processes and explores various numerical methods for solving the governing equations. The model is based on Darcy's law, continuity equation, and constitutive relations of liquid solutions in porous media. Numerical methods, including finite difference, finite element, and finite volume approaches, are discussed with applications to various filtration scenarios. We also provide analysis of the stability, convergence, and efficiency of these methods.

KEYWORDS

Filtration processes, liquid solutions, mathematical model, Darcy's law, numerical methods, finite difference, finite element, finite volume, porous media.

INTRODUCTION

Filtration processes are critical for understanding fluid transport through porous media. These processes have broad applications, from the extraction of hydrocarbons from oil reservoirs to the treatment of polluted groundwater and industrial filtration systems. Understanding how liquids move through porous structures requires robust mathematical models and efficient numerical methods to solve them.

This article aims to formulate a mathematical model for the filtration of liquid solutions through porous media, develop appropriate boundary and initial conditions, and implement numerical methods to solve the model. The focus is on filtration in homogeneous and heterogeneous media, considering both isotropic and anisotropic cases.

The fundamental equation governing the filtration of liquids in porous media is Darcy's Law, which describes the flow of a fluid through a porous material. For incompressible, Newtonian

$$\mathbf{v} = -\frac{k}{\mu}\nabla P$$

fluids, Darcy's law is given by:

where:

- $\mathbf{v}$ is the Darcy velocity (specific discharge),
- k is the permeability of the porous medium,
- μ is the dynamic viscosity of the fluid,
- P is the pressure field.

The mass conservation of an incompressible fluid in a porous medium is expressed by the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

For incompressible flows, this reduces to:

$$\nabla \cdot \left(\frac{k}{\mu}\nabla P\right) = 0$$

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where ρ\rhoρ is the density of the fluid.

The governing equations for the filtration process are obtained by combining Darcy's law and the continuity equation. For a single-phase, incompressible fluid in a porous medium, the resulting partial differential equation (PDE) for pressure PPP is:

$$\frac{d^2P}{dx^2} \approx \frac{P_{i+1} - 2P_i + P_{i-1}}{(\Delta x)^2}$$

This equation is supplemented by appropriate boundary conditions, which may include Dirichlet, Neumann, or mixed conditions depending on the specific problem being modeled.

To solve the governing PDEs for filtration, various numerical methods are employed. The choice of method depends on the complexity of the porous structure, the nature of the boundary conditions, and the required accuracy.

The finite difference method approximates derivatives using discrete differences on a grid. For example, the second-order accurate central difference approximation for the spatial derivative in one dimension is:

$$\int_\Omega \left(\frac{k}{\mu} \nabla P \cdot \nabla v \right) d\Omega = 0$$

In multidimensional problems, finite difference schemes are applied to each direction independently. FDM is suitable for structured grids and simple geometries.

The finite element method divides the computational domain into smaller elements and uses interpolation functions (usually polynomials) to approximate the solution within each element. FEM is particularly advantageous for complex geometries and heterogeneous materials. The weak form of the governing equations is derived, and the solution is obtained by minimizing the residual over the entire domain.

The variational form of the pressure equation in FEM is:

$$\int_\Omega \left(\frac{k}{\mu} \nabla P \cdot \nabla v \right) d\Omega = 0$$

where v is the test function, and Ω represents the computational domain

The finite volume method is widely used in engineering applications because it ensures local conservation of mass and other conserved quantities. In FVM, the domain is divided into control volumes, and fluxes are calculated across the boundaries of these volumes.

The discretized form of the governing equation for each control volume

$$\sum_{\text{faces}} \mathbf{F} \cdot \mathbf{n} = 0$$

Numerical methods for solving PDEs must be stable and convergent to ensure that the solutions are physically meaningful. Stability analysis often involves the Courant–Friedrichs–Lewy (CFL) condition, which provides a criterion for the time step in explicit time-stepping schemes.

For implicit methods, stability is generally guaranteed, but they require solving large linear systems at each time step. Convergence refers to the behavior of the numerical solution as the grid is refined. A method is said to converge if the solution approaches the true solution as the grid spacing decreases.

To demonstrate the application of the developed mathematical model and numerical methods, we consider a case study of fluid filtration through a heterogeneous porous medium. The permeability field is assumed to vary spatially according to:

$$k(x, y) = k_0 \exp\left(-\frac{x^2 + y^2}{L^2}\right)$$

is the reference permeability, and L is the characteristic length scale of the heterogeneity. The finite element method is used to solve the pressure distribution in the domain, and the results are compared with experimental data.

The mathematical model and numerical methods developed in this article provide a framework for simulating filtration processes of liquid solutions in porous media. The choice of numerical method depends on the geometry of the domain, material properties, and desired accuracy. While finite difference methods are efficient for simple geometries, finite element and finite volume methods offer greater flexibility and accuracy for complex systems.

Future research may focus on extending the model to account for multi-phase flows, reactive transport, and the coupling between mechanical deformation and fluid flow in deformable porous media.

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