American Journal Of Applied Science And Technology
(ISSN – 2771-2745)
VOLUME 04 ISSUE 06 Pages: 76-81
OCLC – 1121105677

Publisher: Oscar Publishing Services

Journal Website: https://theusajournals.com/index.php/ajast

Research Article

APPLICATION OF DIFFERENTIAL EQUATIONS IN VARIOUS FIELDS OF SCIENCE

Submission Date: June 20, 2024, Accepted Date: June 25, 2024,
Published Date: June 30, 2024
Crossref doi: https://doi.org/10.37547/ajast/Volume04Issue06-15

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ABSTRACT

The article, "Application of Differential Equations in Various Fields of Science," explores the use of differential equations for modeling economic and natural phenomena. It examines two main models of economic dynamics: the Evans model for the market of a single product, and the Solow model for economic growth.

The author emphasizes the importance of proving the existence of solutions to differential equations in order to verify the accuracy of mathematical models. They also discuss the role of electronic computers in developing the theory of differential equations and its connection with other branches of mathematics such as functional analysis, algebra, and probability theory.

Furthermore, the article highlights the significance of various solution methods for differential equations, including the Fourier method, Ritz method, Galerkin method, and perturbation theory.Special attention is paid to the theory of partial differential equations, the theory of differential operators, and problems arising in physics, mechanics, and technology. Differential equations are the theoretical foundation of almost all scientific and technological models and a key tool for understanding various processes in science, such as in physics, chemistry, and biology.

Examples of processes described by differential equations include normal reproduction, explosive growth, and the logistic curve. Cases of using differential equations to model deterministic, finite-dimensional, and differentiable phenomena, as well as the impact of catch quotas on population dynamics, are discussed.

In conclusion, the significance of differential equations for research and their role in stimulating the development of new mathematical areas is emphasized.

KEYWORDS

Differential equations, mathematical modeling, economic dynamics, the Evans model, the Solow model, the theorems on the existence of solutions, evolutionary processes, the Fourier method, the Ritz method, the Galerkin method, partial differential equations, the averaging theory, physics, chemistry, biology, the logistic curve, catch quotas, the theory of differential operators, the explosion equation, and normal reproduction.

INTRODUCTION

Mathematical modeling of economic and natural processes leads to the need to solve equations that, in addition to independent variables and the desired functions dependent on them, also contain derivatives or differentials of unknown functions. Such equations are called differential.

Differential equations are widely used in models of economic dynamics, in which they study not only the dependence of variables on time, but also on their relationship in time. Such models are: the Evans model - establishing a balanced price in the market for one product; as well as a dynamic model of economic growth, known as the "basic Solow model".

In the Evans model, the market for a single product is considered, time is considered continuous. Let $d(t),\ s(t),\ p(t)$ - demand, supply and price according to this product at a time. Suppose that supply and demand are linear functions of price, that is, $d(p) = a - bp$, $a, b > 0$ – demand decreases with increasing prices, and, $s(p) = \alpha + \beta p$, $\alpha, \beta > 0$ – supply increases with increasing prices. The ratio $a > \alpha$ is natural, that is, at zero price, demand exceeds supply.

The Solow model considers the economy as a whole (without structural units). This model adequately reflects the most important macroeconomic aspects of the production process.

It is important to note that to verify the correctness of the mathematical model, the existence theorems of solutions to the corresponding differential equations are very important, since the mathematical model is not always adequate to a specific phenomenon and the existence of a solution to a real problem (physical, chemical, biological) does not imply the existence of a solution to the corresponding mathematical problem.

Currently, the use of modern electronic computers plays an important role in the development of the theory of differential equations. The study of differential equations often makes it easier to conduct a computational experiment to identify certain properties of their solutions, which can then be theoretically justified and will serve as the foundation for further theoretical research.

So, the first feature of the theory of differential equations is its close connection with applications. In other words, we can say that the theory of differential equations was born from applications.

The second feature of the theory of differential equations is its connection with other branches of mathematics, such as functional analysis, algebra, and probability theory.

In the study of specific differential equations that arise in the process of solving physical problems, methods are often created that have great commonality and are applied without a rigorous mathematical justification to a wide range of mathematical problems. Such methods are, for example, the Fourier method, the Ritz method, the Galerkin method, perturbation theory methods, and others. The effectiveness of the

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Crossref doi Google Scholar WorldCat MENDELEY
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application of these methods was one of the reasons for the attempts of their rigorous mathematical justification. This led to the creation of new mathematical theories, new areas of research.

Currently, the theory of partial differential equations is a rich, highly branched theory. The theory of boundary value problems for elliptic operators is constructed on the basis of a recently created new apparatus - the theory of pseudo-differential operators, the index problem is solved, and mixed problems for hyperbolic equations are studied.

In recent decades, a new branch of the theory of partial differential equations has arisen and is developing intensively - the theory of averaging of differential operators. This theory arose under the influence of the problems of physics, continuum mechanics and technology, in particular, related to the study of composites (highly heterogeneous materials widely used at present in engineering), porous media, and perforated materials.

Thus, differential equations are currently a complex collection of facts, ideas, and methods that are very useful for applications and stimulating theoretical research in all other branches of mathematics. Many sections of the theory of differential equations have grown so much that they have become independent sciences. It can be said that most of the paths connecting abstract mathematical theories and natural science applications go through differential equations.

Differential equations are the theoretical basis of almost all models used in science and technology. Such processes are reflected in physics, chemistry, biology and almost all areas of science. Almost all problems of physics lead to the need to solve differential equations. This is due to the fact that virtually all physical laws that describe physical phenomena are differential equations for certain functions that describe these phenomena. Such physical laws are a theoretical generalization of numerous experiments and characterize the evolution of the sought quantities in the general case, both in space and in time. The solution of differential equations is a key task for many areas of human activity, and also plays an important role in the knowledge of the world.

The theory of ordinary differential equations is one of the main tools of mathematical natural science. This theory allows us to study all kinds of evolutionary processes with the properties of determinism, finite-dimensionality and differentiability. Before giving exact mathematical definitions, consider a few examples.

1. Examples of evolutionary processes. A process is called deterministic if its entire future course and its entire past are uniquely determined by the current state. Many of the various states of the process are called phase space. So, for example, classical mechanics considers the movement of systems whose future and past are uniquely determined by the initial positions and initial velocities of all points of the system. The phase space of a mechanical system is a set whose element is a set of positions and velocities of all points of a given system. Particle motion in quantum mechanics is not described by a deterministic process. Heat distribution is a semi-deterministic process: the future is determined by the present, but the past is not. A process is called finite-dimensional if its phase space is finite-dimensional, that is, if the number of parameters needed to describe its state is finite. So, for example, Newtonian mechanics of systems from a finite number of material points or solids belongs to this class. The dimension of the phase space of a system of material points is 6n, and that of a system of n solids is –– . Fluid motions studied in

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hydrodynamics, processes of string and membrane vibrations, wave propagation in optics and acoustics are examples of processes that cannot be described using finite-dimensional phase space.

A process is called differentiable if its phase space has the structure of a differentiable manifold, and a change in state over time is described by differentiable functions. So, for example, the coordinates and speeds of the points of a mechanical system change over time in a differentiable way.

The movements studied in shock theory do not possess the differentiability property.

Thus, the motion of a system in classical mechanics can be described using ordinary differential equations, while quantum mechanics, the theory of thermal conductivity, hydrodynamics, the theory of elasticity, optics, acoustics, and impact theory require other means.

Two more examples of deterministic finite-dimensional and differentiable processes: the process of radioactive decay and the process of reproduction of bacteria with a sufficient amount of nutrient. In both cases, the phase space is one-dimensional: the state of the process is determined by the amount of substance or the number of bacteria. In both cases, the process is described by an ordinary differential equation.

We note that the form of the differential equation of the process, as well as the very fact of the determinism, finite-dimensionality, and differentiability of a particular process can only be established experimentally, therefore, only with a certain degree of accuracy. In the future, we will not emphasize this circumstance every time and will talk about real processes as if they exactly coincided with our idealized mathematical models.

2. An evolution equation with one-dimensional phase space. Consider the equation

$$\dot{x} = v(x), \ \ x \in \square \ . \qquad (1)$$

This equation describes an evolutionary process with one-dimensional phase space. The right side defines the vector field of the phase velocity: a vector is applied at the point. Such an equation, the right side of which is independent of, is called autonomous. The evolution rate of an autonomous system, that is, a system that does not interact with others, is determined only by the state of this system: the laws of nature do not depend on time.

The solution to this equation is given by the formula

$$t - t_0 = \int_{x_0}^{x} \frac{d\xi}{v(\xi)} \qquad (2)$$

3. Example: the equation of normal reproduction. Suppose that the size of the biological population (for example, the number of bacteria in a Petri dish or fish in a pond) is equal and that the growth rate is proportional to the number of individuals present. The breeding equation (This assumption is approximately satisfied, while there is a lot of food.)

Our assumption is expressed by the differential equation of normal reproduction

$$\dot{x} = kx, \ \ k > 0 \qquad (3)$$

According to the meaning of the problem $x > 0$, so that the direction field is specified in the half-plane; It is clear from the form of the direction field that it x grows with growth t, but it is not clear whether the infinite values x will be reached in a finite time (the vertical asymptote of the integral curve t) or does the

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solution remain finite for all? Along with the future, the past is also unclear: will the integral curve tend to the axis $x=0$ while striving for a finite negative limit t or an infinite?

Fortunately, the breeding equation is solved explicitly.

$$t-t_0=\int_{x_0}^{x}\frac{d\xi}{k\xi},\quad k(t-t_0)=\ln\frac{x}{x_0},\quad x=e^{k(t-t_0)}x_0$$

(4)

This theorem was discovered by Barrow precisely when solving the simplest differential equations, now called equations with separable variables. Therefore, the solutions of the normal multiplication equation grow exponentially at $t\to+\infty$ and exponentially decrease at $t\to-\infty$; neither infinite nor zero values x are reached at finite t. To double the population according to the equation of normal reproduction, it is therefore always necessary the same time, regardless of its number.

4. Example: explosion equation. Now suppose that the growth rate is proportional not to the number of individuals, but to the number of pairs:

$$\dot{x}=kx^2. \qquad (5)$$

In this case, with large growths it is much faster than normal, and with small growths it is much slower (this situation is more likely to occur in physicochemical problems, where the reaction rate is proportional to the concentrations of both reagents; however, it is now so difficult for some whales to find a mate that whale breeding obeys equation (5), moreover, a little).

The field of directions does not seem to differ much from that for ordinary reproduction, but calculations show that the integral curves behave completely differently. Assume for simplicity that $k=1$. Using Barrow's formula, we find a solution $t=\int\frac{dx}{x^2}+C$, that is $x=-\frac{1}{t-C}$ at $t<C$, that is at. Integral curves - half hyperbole. Hyperbola has a vertical asymptote.

So, if population growth is proportional to the number of pairs, then the number of population becomes infinitely large in a finite time. Physically, this conclusion corresponds to the explosive nature of the process. (Of course, at if too close to C, the idealization adopted in the description of the process by the differential equation is not applicable, so that the real number of people in a finite time does not reach infinite values.).

5. Example: logistic curve. The usual breeding equation is suitable only as long as the number of individuals is not too large. With an increase in the number of individuals, competition due to food leads to a decrease in the rate of growth. The simplest assumption is that the coefficient k depends on x how a linear inhomogeneous function (if not too large, any smooth function can be approximated by a linear inhomogeneous function) : $k=a-bx$. Thus we come to the competition equation taking into account competition $\dot{x}=(a-bx)x$. The coefficients a and b can be turned into a unit by choosing the scales t and x. We get the so-called *logistic equation*.

$$\dot{x}=(1-x)x.$$

The vector field of the phase velocity v and the field of directions on the plane (t,x). We see that

1) the process has two equilibrium positions: $x=0$ and $x=1$;
2) between points 0 and 1, the field is directed from 0 to 1, and at –– to point 1. Thus, the equilibrium position 0 is unstable (once the population appears begins to grow), and the equilibrium position 1 is stable (a smaller population grows, and a larger population decreases). Whatever the initial state $x>0$, over time, the process reaches a steady state of equilibrium $x=1$. From these considerations, it is not clear, however, whether this exit occurs in a finite or in infinite time, i.e., do the integral curves that begin in the region $0<x<1$ have common points with a straight line $x=1$? It can be shown that there are no such common points and that these integral curves asymptotically tend to the line $x=1$ at $t \to +\infty$ and to the line $x=0$ for $t \to -\infty$. These curves are called logistic curves. Thus, the logistic curve has two horizontal asymptotes ($x=0$ and $x=1$) and describes the transition from one state (0) to another (1) in infinite time.

6. Example: catch quotas. So far, we have considered a free population developing according to its internal laws. Suppose now that we catch part of the population (say, we fish in a pond or in the ocean). Assume that the catch rate is constant. We arrive at a differential catch equation $\dot{x}=(1-x)x-c$. The value c characterizes the catch rate and is called the quota. We see that at a not too high catch rate ($0<c<\frac{1}{4}$), there are two equilibrium positions.

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