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Basic Concepts of Dynamical Systems and Their Analysis in Examples

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Abstract: This article provides an introduction to the concept of dynamical systems and explores several fundamental notions, such as iterative processes, the orbit of an initial point, and fixed points. Along with theoretical definitions, a detailed analysis of these concepts is presented through various examples. Furthermore, to facilitate the learning process for undergraduate and graduate students entering the field of dynamical systems, each foundational concept is thoroughly explained using illustrative examples.

Keywords: Dynamical system, iteration, orbit of an initial point, fixed point.

INTRODUCTION:

To understand the meaning of a dynamic system clearly, let's consider the following situation. Suppose we have a deposit of 1 million in a bank with a 20% interest rate. The question arises: if we use this money n If we do not touch it for a year, how much money will be in our account at the end of this period? For simplicity, the 20 percent interest is added to our account once at the end of each year and at the end of the next year the interest is added to the money that was added. This corresponds to the problem of compound interest known to us from the 6th grade mathematics course. This is one of the simplest examples of an iterative (repeating) process or dynamic system.

Now we find the solution to the above problem.

That is, the amount we are currently depositing x_1 Now, at the end of the first year, 20 percent of this amount is added to the amount, and $x_2 = 1.2x_1$ A sum of money is generated in the amount of a unit of money. This sum of money for the following year $x_3 = 1.2x_2 = 1.2^2 x_1$ A sum of money is generated. By repeating this process repeatedly, n amount of money collected during the period $x_n = 1.2x_{n-1} = 1.2^2 x_{n-2} = ... = 1.2^{n-1} x_1$ It can be derived that is equal to. This is an iterative process, as we have noted above. Or it can be understood as an introduction to dynamic systems.

The above problem is an economic problem for developing the concept of dynamic systems. In fact, such problems exist not only in economics but also in nature. For example, suppose we want to predict how a population of a certain species will behave, increasing or decreasing during the change of generations. n -the population that survived in the generation P_n Let's denote it by . So, our question is as follows: n increasingly P_n Can we predict what will happen with? P_n Will the species go extinct, going to zero? Or P_n Will there be a "population explosion" that will grow indefinitely?

There are many mathematical models for predicting population behavior. The simplest (and most straightforward) of these is the exponential growth model. In this model, we assume that the population of the next generation is directly proportional to the population of the current generation. This is translated into mathematical language as the following differential equation:

$$P_{n+1} = rP_n$$

Here r —a fixed number determined by environmental conditions. Thus, the initial population P_0 Given, we can recursively determine the population of the next generations:

$$P_1 = rP_0$$
$$P_2 = rP_1 = r^2P_0$$
$$P_3 = rP_2 = r^3P_0$$
$$\vdots$$
$$P_n = rP_{n-1} = r^nP_0$$

As before, we determine the state of the population by iteration. In this case, the function we are iterating over is $F(x) = rx$ It will look like this. So:

$$P_n = \underbrace{F\circ\ldots\circ F}_{n \text{ marta}}(P_0) = r^nP_0$$

Note that the ultimate fate of the population r depends on the coefficient. If $r > 1$ if, then n with the increase r^n tends to infinity, which means we have uncontrolled population growth. If $r < 1$ if, r^n tends to zero, resulting in the species becoming extinct. Finally, if $r = 1$ if, $P_n = P_0$ will be, that is, there will be no change in the population. Thus, we can assume that any r we can achieve our goal of determining the fate of a species. Of course, this simplified model is not very realistic, because real-life populations behave in much more complex ways. For example, populations can never go to infinity.

As we have seen above, there are many types of problems in science and mathematics that involve iteration. Iteration means repeating a process over and over. In dynamics, this repeated process is the application of a function.

Iterating a function means repeatedly calculating the function, using the previous result as the input for the next step. In mathematical terms, this is the process of repeatedly composing a function with itself (forming a more complex function).

We write this as follows. F for the function $F^2(x) - F$ The second iteration of, i.e. $F(F(x)), F^3(x)$ — third iteration $F(F(F(x)))$ and in general, $F^n(x) - F$ with himself n times composition. For example, if $F(x) = x^2 + 1$ if so, then:

$$F^2(x) = (x^2+1)^2 + 1$$
$$F^3(x) = ((x^2+1)^2+1)^2 + 1.$$

Likewise, if $F(x) = \sqrt{x}$ if so, then:

$$F^2(x) = \sqrt{\sqrt{x}}$$
$$F^3(x) = \sqrt{\sqrt{\sqrt{x}}}.$$

It is very important to understand that, $F^n(x)$ sign $F(x)$ does not mean raising the function to the \$n\$-th power (we never use this operation). On the contrary, $F^n(x)$ — that x calculated at the point F function n **-is the chi iteration.**

Now let's get acquainted with the concept of the orbit of dynamics. $x_0 \in \square$ if given, x_0 of the point F The orbit under the function is called $x_0, x_1 = F(x_0), x_2 = F^2(x_0), \ldots, x_n = F^n(x_0), \ldots$ is called a sequence of points. The point is called the seed of the orbit.

For example, if $F(x) = \sqrt{x}$ if and $x_0 = 256$ If we take, the first few points of the orbit will be as follows:

$$x_0 = 256$$
$$x_1 = \sqrt{256} = 16$$
$$x_2 = \sqrt{16} = 4$$
$$x_3 = \sqrt{4} = 2$$
$$x_4 = \sqrt{2} = 1.41\ldots$$

Example: $F(x) = x^2$ **Given a function.** $\frac{1}{2}$ Calculate the first five points of the orbit of the point.

Solution: As we noted above, in the given problem, we iterate over the values of the function five times to find the first five orbits of the function.

$$x_0 = \frac{1}{2},\ F_1(x) = F^2(x) = \left(\frac{1}{2}\right)^2 = \frac{1}{4},\ F_2(x) = F^3(x) = (F_1(x))^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16},$$
$$F_3(x) = \left(\frac{1}{16}\right)^2 = \frac{1}{256},\ F_4(x) = \left(\frac{1}{256}\right)^2 = \frac{1}{65536},\ F_5(x) = \left(\frac{1}{65536}\right)^2 = \frac{1}{4294967296}$$

Example: $F(x) = x^2 - 2$ Given a function. $F^2(x)$ and $F^3(x)$ Calculate the.

Solution: As can be seen from the example condition, we need to find the second and third order iterations of the given function.

$$F_1(x) = F(x) = x^2 - 2,\ F_2(x) = F^2(x) = (x^2-2)^2 - 2 = x^4 - 4x^2 + 2,$$
$$F_3(x) = F^3(x) = (x^2-2)^4 - 4(x^2-2)^2 + 2 = x^8 - 8x^6 + 24x^4 - 32x^2 + 16 - 4x^4 + 16x^2 - 16 + 2 =$$
$$= x^8 - 8x^6 + 20x^4 - 16x^2 + 2$$

From the above, it can be seen that the orbits of the given function up to the third iteration are as follows.

$$x^2 - 2,\quad x^4 - 4x^2 + 2,\quad x^8 - 8x^6 + 20x^4 - 16x^2 + 2$$

Example: $F(x) = x^2$ Given a function. $F^2(x),\ F^3(x)$ and $F^4(x)$ Calculate the functions. $F^n(x)$ What is the general formula for?

Solution: $F^2(x),\ F^3(x)$ and $F^4(x)$ To find the values, we perform the same calculations as in the previous examples.

$$F_1(x) = F(x) = x^2,\ F_2(x) = F^2(x) = F(F(x)) = (x^2)^2 = x^4,$$
$$F_3(x) = F^3(x) = F(F(F(x))) = x^8,\ F_4 = F^4(x) = x^{16}$$

Equalities are formed, and from these equalities, $F^n(x)$ The following general formula can be derived for

$$F_n(x) = F^n(x) = F(F(F(...F(x)))) = x^{2^n}$$

Now we will get acquainted with the concept of a fixed point, which is one of the important concepts of orbits in dynamical systems.

In a typical dynamical system, there are many different types of orbits. Without a doubt, the most important type of orbit is the fixed point. A fixed point is $F(x_0) = x_0$ satisfying the condition x_0 is said to be a point. Note that, $F^2(x_0) = F(F(x_0)) = F(x_0) = x_0$ and in general, $F^n(x_0) = x_0$. Thus, the orbit of the fixed point $x_0, x_0, x_0, \ldots$ consists of a fixed sequence of the form . A fixed point never "moves". As the name suggests, it is fixed by a function. For example, 0,1 and -1 all of them $F(x) = x^3$ are fixed points for the function, $F(x) = x^2$ and for the function only 0 and 1 is a fixed point.

Fixed points are found by solving the following equation:

$$F(x) = x.$$

Example: $F(x) = x^3 - 3x$ find the fixed points of the function.

Solution: As we mentioned above, to find the fixed points of the function $F(x) = x$ we find the solution to the equation.

$$x^3 - 3x = x$$
$$x^3 - 4x = 0$$
$$x(x^2 - 4) = 0$$
$$x_1 = 0,\ x_2 = -2,\ x_3 = 2$$

It can be seen that this function has three fixed points, namely $x_1 = 0,\ x_2 = -2,\ x_3 = 2$

Example: $F(x) = x\sin x$ Find all real fixed points of the function.

Solution: To find the solution to this example, we use the same method as in the previous examples. $F(x) = x$ we find the solution to the equations.

$$x\sin x = x$$
$$x\sin x - x = 0$$
$$x(\sin x - 1) = 0$$
$$x_1 = 0,\ \sin x = 1$$
$$x_1 = 0,\ x = \frac{\pi}{2} + 2\pi n$$

As can be seen from the solutions of the equation, there are infinitely many fixed points of this function.

CONCLUSION

This article introduces the concept of dynamical systems and introduces several basic concepts of dynamical systems (for example: iterative process, initial point orbit and fixed point), examines their analysis with examples, and also studies the analysis of these concepts in examples. The main goal of this is to create convenience for new students of the field of dynamical systems and help them visualize the basic concepts, and we believe that the goal set in this article has been achieved.

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