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Mathematical Model Of Bank Erosion And Horizontal Deformations In The Riverbed

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Abstract: This article presents a mathematical model describing the dynamics of bank erosion and horizontal deformations in the lower reaches of the Amu Darya River. The study focuses on the development of the erosion boundary along straight and curved riverbank sections. The process is analyzed as a time-dependent phenomenon characterized by varying intensities and stabilization phases. Empirical coefficients were introduced to express the relationship between the length and width of the erosion zone, enabling the determination of its geometric and temporal parameters. The model provides analytical expressions that characterize the shape and development rate of the eroded area, incorporating the asynchronous onset of stabilization for both longitudinal and transverse dimensions. The proposed theoretical formulations were verified using field data collected from the lower Amu Darya, showing an accuracy of 5–10%, which confirms the adequacy of the developed model for engineering applications.

Keywords: Riverbank erosion, deformation intensity, erosion boundary, curved riverbank, empirical coefficient, dimensionless time, stabilization process.

Introduction: Riverbank erosion and horizontal channel deformation are among the most significant processes affecting river morphology, sediment transport, and hydraulic stability, particularly in large alluvial rivers like the Amu Darya. These processes are particularly active in sections where the flow structure interacts with easily erodable banks, leading to continuous changes in the plane geometry and the displacement of the flow axis.

In the lower reaches of the Amu Darya, regulated flow conditions resulting from hydraulic structures such as the Tuyamuyin reservoir and the Takhiya-Tash hydro complex have significantly altered the natural channel dynamics. Under such regulated regimes, understanding and predicting local bank erosion patterns becomes essential for ensuring the stability of protective embankments and river regulation works.

Previous research on riverbank erosion has largely focused on empirical or semi-empirical formulas, which often neglect the time-dependent evolution and the asymmetrical nature of the erosion boundary. In this study, a mathematical model is developed to describe the erosion process in both straight and curved riverbank sections, taking into account the temporal variation in erosion intensity and the asynchronous stabilization between longitudinal and transverse directions.

METHODS

The study employed a combined analytical and empirical modeling approach to describe the process of riverbank erosion and horizontal deformation in the lower reaches of the Amu Darya River. The modeling framework was developed based on geometric representation of the erosion boundary and the time-dependent variation of deformation intensity.

The erosion boundary in straight and curved riverbank sections was expressed using parabolic and symmetrical equations that describe the planar deformation of the eroded area. For straight banks, the shape of erosion was modeled as the sum of two parabolic functions representing symmetrical displacement towards the bank. The relationship between erosion length-L and width-B was defined by analytical equations (3)-(4), which allow determining the geometry of the eroded zone.

The proposed model is applicable for assessing local erosion near hydraulic structures, designing bank protection systems, and forecasting deformation zones in regulated river conditions. The methodology provides a theoretical basis for improving hydraulic resistance assessments and optimizing river regulation practices in the Amu Darya basin.

It is known that the process of local erosion is observed on straight and curved coasts. In this case, the intensity of deformation is determined by the rate of displacement of the erosion boundary towards the shore.

RESULTS

The equation of the circular erosion boundary formed on a rectilinear coastal section is obtained by adding two symmetrical parabolas (Fig. 1).

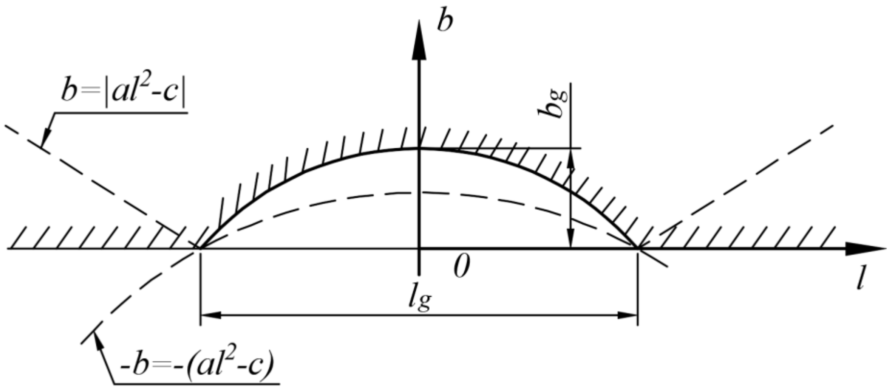

Figure 1. Scheme for deriving the equation of the form of erosion of a rectilinear bank.

$$b = |al^2 - c| - (al^2 - c), \quad (1)$$

here: $a > 0$ is the degree of curvature of the parabola's curve, and $c > 0$ is the height or half-deepness at the center of the parabola.

Here, $l_g = l$ —the length of the erosion zone, $b_g = b$ —the width of the erosion zone . At the center of symmetry $l = 0$ that is, this will be the deepest (highest) point. Using this condition, we substitute 0 for l in formula (1) and obtain.

$$b = | a \cdot 0^2 - c | - (a \cdot 0^2 - c),$$

$$b = | -c | - (-c) = c + c = 2c.$$

From this, the depth of the center,

$$b_g = 2c \quad (2)$$

we obtain the formula.

Now we find l from the condition that $b = 0$ must be at the edge of the washing point.

$$0 = | al^2 - c | - (al^2 - c)$$

To make this expression equal to zero:

$$| al^2 - c | = (al^2 - c)$$

equality must be valid. This equality holds when

$$al^2 - c \geq 0$$

if. From this

$$al^2 - c = 0, \text{ va } al^2 = c \text{ yoki } l = \sqrt{\frac{c}{a}},$$

here: $l = \sqrt{\frac{c}{a}}$ is the half-length.

So, half length

$$l = \sqrt{\frac{c}{a}}.$$

Since the region is symmetrical, the total length is equal to.

$$l_g = 2\sqrt{\frac{c}{a}}.$$

For the length of the erosion zone l_g and the width b_g, we obtain the following equalities.

$$l_g = 2\sqrt{\frac{c}{a}}, \qquad (3)$$

$$b_g = 2c. \qquad (4)$$

In the diagram, the linear expression below (---), $-b = -(a\,l^2 - c)$ is simply drawn to show mathematical symmetry.

It is known that the shift of the erosion boundary towards the shore is a decreasing process over time, the intensity of which is not the same at different stages of time (from the beginning of erosion to stabilization). The process of formation along the ob axis of the dimensionless (η_l)) erosion limit (width) of local deformation can be described by the following expression.

$$\eta_l = \eta_t \cdot K_1^{\eta_t - 1}, \qquad (5)$$

here, $\eta_t = \frac{t}{t_{cm}}$- dimensionless deformation time,

K_1=ob is an empirical coefficient that determines the individual nature of the deformation process along the axis.

Similarly, the process of formation along the ol axis of the dimensionless (η_b) erosion limit (width) of local deformation can be described by the following expression.

$$\eta_b = \eta_t \cdot K_2^{\eta_t - 1}, \qquad (6)$$

where K_2=ol is the empirical coefficient determining the individual nature of the axial deformation process.

In formulas (5) and (6) $\eta_l = \frac{l}{l_{cm}}$ and $\eta_b = \frac{b}{b_{cm}}$ are dimensionless values of coastal erosion in plan, determined by the ratio of the length or width of the erosion at time t to the corresponding values for the period of erosion stabilization t_cm.

This form of describing the washing process over time corresponds to the results of natural and laboratory experiments. In this case, the synchronousness of the initial stabilization moment along the width and length of the erosion section is taken into account (K_1, K_2 va η_{t_l}, η_{t_b}). The values of the empirical coefficients K_1, K_2 are 0,3-0,5. This process is graphically represented in Fig. 2.

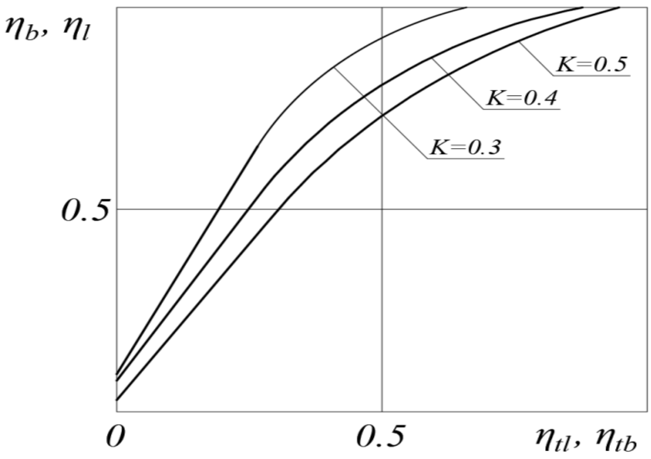

Figure 2. Dependence of the amount of erosion on time in dimensionless coordinates.

The characteristic integral equation can be used as an additional parameter for describing the process under consideration.

$$D = \int_0^1 \eta_{t_{l,b}} \cdot K_{1,2}^{\eta_t - 1} \cdot d_{l,b} \qquad (3.48)$$

this formula is determined by the area of the curve $\eta_{l,b}(\eta_{t_{l,b}})$ on the interval (0,1). To make it convenient to select the coefficients K_1, K_2 in the table. Fig. 3 shows the values of the function $0{,}2 \leq \eta_{t_{l,b}} \leq 0{,}8$ (it is clear that $\eta_{t_{l,b}} = 0$ when $\eta_{l,b} = 0$ and $\eta_{l,b} = 1$ when $\eta_{t_{l,b}} = 1$, as well as the values of D).

Returning to the description of the erosion limit, considering the above, we determine the coefficients a and c of equation (1).

Using the above conditions, that is, using $l = 0$ and $b = 0$, we find (1):

$$c = 0{,}5 b_g \qquad (8)$$

and

$$b_g = \eta_b \cdot b_{g.st} = \eta_{t_b} \cdot K_2^{\eta_t - 1} \cdot b_{g.st} \qquad (9)$$

$$c = 0{,}5 \eta_{t_b} \cdot K_2^{\eta_t - 1} \cdot b_{g.st} \qquad (10)$$

from formula (1)

$$a = 2 l_g b_g \qquad (11)$$

but

$$l_g = \eta_{t_b} \cdot K_1^{\eta_{tl} - 1} \cdot l_{g.st}, \qquad (12)$$

then

$$a = 2 b_{g.st} \cdot l_{g.st}^{-2} \cdot \eta_{t_b} \cdot \eta_{t_l}^{-2\left(1 - \eta_{t_b}\right)} \cdot K_2^{\eta_{t_b} - 1} \qquad (13)$$

using these, we can write (1) as follows.

$$b = a\left[\left|l^2 - \frac{c}{a}\right| - (l^2 - \frac{c}{a})\right], \qquad (14)$$

where: Substituting the coefficients a and c from (9) and (12), we obtain:

$$b = 0{,}5 \cdot b_{g.st} l_{g.st}^{-2} \cdot \eta_{t_b} \cdot \eta_{t_l}^{-2} \cdot K_1^{-2(\eta_{t_b} - 1)} \cdot \left[\left| l^2 - \left(l_{g.st} \cdot \eta_{t_l} \cdot\cdot K_1^{(\eta_{t_l} - 1)}\right)^2 \cdot \frac{1}{4}\right| - l^2 + \left(l_{g.st} \cdot \eta_{t_l} \cdot K_1^{(\eta_{t_l} - 1)}\right)^2 \cdot \frac{1}{4}\right]. \qquad (15)$$

DISCUSSION

Thus, equations (12) and (15) were obtained, which determine the shape of the boundary of intensive local erosion in the rectilinear part of the bank relative to the regulated conditions of water flow in the Amu Darya, taking into account the asynchronousness of the time of the beginning of stabilization (η_{t_l}, η_{t_b}) and the different nature of the local deformation process during the time from the beginning of stabilization (K_1, K_2) in the direction of the width and length (ob, ol) of the deformation zone.

Table 1.

The values of the function $\eta_{l,}\ \eta_b$ and the parameter D.

K_1, K_2	η_{t_l}, η_{t_b}				D
	0,20	0,40	0,60	0,80	
0,300	0,52	0,82	0,97	1,00	0,7791
0,325	0,49	0,78	0,94	1,00	0,7544
0,350	0,46	0,75	0,91	0,99	0,7325
0,375	0,44	0,72	0,89	0,97	0,7129
0,400	0,42	0,69	0,87	0,96	0,6952
0,425	0,40	0,67	0,84	0,95	0,6792
0,450	0,38	0,65	0,83	0,94	0,6645
0,500	0,35	0,61	0,79	0,92	0,6387

To express the shape of coastal erosion, it is necessary to define the erosion equation. Let us assume that the ordinary line of the bank is drawn along a parabola of the following form.

$$b = -dl^2, \quad (16)$$

here: d is a positive number.

Combining the given formula with (1), we obtain the desired formula (Fig. 3).

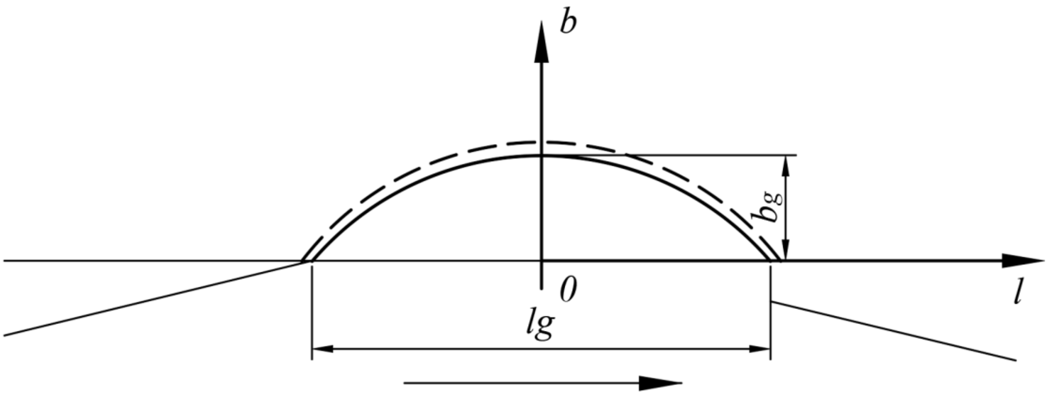

Figure 3. Scheme for deriving the equation of the form of erosion.

Thus, using formulas (2), (10) and (13), it is possible to determine the parameters of the local erosion zone. The parameter d is determined, for example, based on the radius of curvature K_2 of the given coastline:

$$K_2 = \frac{\frac{d^2B}{dl^2}}{\left[1 + (\frac{dB}{dl})^2\right]^{\frac{3}{2}}} = -\frac{2d}{(1 - 4d^2l^2)^{\frac{3}{2}}} \quad (3.58)$$

Thus, the desired equation takes the following form:

$$b = |al^2 - c| - [(a + d)l^2 - c]. \quad (3.59)$$

coefficients a and c are determined by formulas (10) and (13).

CONCLUSION

It should be noted that equations describing the intensive local deformation processes of the bank, consisting of easily erodable soils, arising as a result of changes in the dynamic axis of the flow in the existing curvilinear channels of various sections of the lower reaches of the Amu Darya, were obtained. Calculations based on the obtained theoretical equations were compared with the results of field measurements. The difference in comparison was 5-10%.

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